ECONtribute Discussion Paper

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August 2020

ECONtribute Discussion Paper No. 021

Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany´s Excellence Strategy – EXC 2126/1– 390838866 is gratefully acknowledged.

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August 6, 2020

Abstract

Many low-wage workers in the United States are subject to non-compete clauses, which forbid them to work for competitors. Empirical research has found a link between the prevalence of non-compete clauses and minimum wage legislation. To explain this link, we propose a moral hazard model with minimum wages. Non-compete clauses can be used to punish failure. We characterize the optimal contracts with and without the possibility to use a non-compete clause. We find that the principal only uses a non-compete clause if minimum wages are sufficiently high. Non-compete clauses transfer utility from the agent to the principal because they increase the equilibrium effort without increasing the wages. If non-compete clauses can be arbitrarily severe, there is no minimum wage for which the agent gets a rent. If non-compete clauses are bounded, both the principal and the agent might be made better off than without non-compete clauses.

JEL classification: D86, J32, J41, K31
Keywords: non-compete clause, minimum wage, limited liability, moral hazard, rent extraction.

*We thank the numerous colleagues that discussed our idea with us and gave us feedback on our drafts. Especially, we thank Jörg Budde, Simon Dato, Oliver Gürtler, Hendrik Hakenes, Carl Heese, Andreas Klümper, Matthias Kräkel, Stephan Lauermann, Helene Mass, Justus Preußer, Tobias Rachidi, Paul Schäfer, Patrick Schmitz, Lennart Struth, and Max Thon. Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC 2126/1– 390838866.

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1 Introduction

A non-compete clause (*NCC*) is part of an employment contract that prohibits employees from working for a competitor or from starting their own business within specific geographic or temporal boundaries. A significant fraction of the US labor force is currently bound by a non-compete clause: 20% of the labor force were restricted by such a clause in 2014 and 40% had signed one in the past (Starr, Prescott, and Bishara, 2019b). Moreover, many low-wage workers are bound by NCCs. Colvin and Shierholz (2019) find that 29% of the sampled workplaces that pay an average hourly salary of less than 13 dollars and 20% of the workplaces in which the typical employee has not graduated from high school have each employee sign an NCC. While the public seems to accept NCCs in the contracts of CEOs, media reports about NCCs in the contracts of low-wage workers caused a public outrage.[1] As a result, there have been several attempts at restricting the use of NCCs, particularly concerning low-wage workers, in the last years.[2] Both the public and the politicians advocating for restrictions believe that NCCs exploit low-wage workers. The exact mechanism, however, has remained unclear.

Our main contribution is showing that effort incentives of NCCs can be such a mechanism. To show this, we use the canonical partial-market moral hazard model and add the possibility to costlessly reduce the agent's payoff with an NCC. The employer terminates the employee after a bad performance, which activates the NCC and restricts the agent's employment possibilities. To avoid this, the agent exerts more effort. Thus, both a bonus wage and an NCC provide incentives—they are substitutes in the incentive constraint. If a sufficiently large minimum wage restricts the principal's use of bonus wages, she resorts to an NCC. The reason for this is that a bonus wage and an NCC are opposites in the participation constraint: The bonus wage makes the participation constraint slack, as it increases the agent's payoff after a good outcome, whereas the NCC makes the participation constraint tight, as it decreases the payoff after a bad outcome. Because the minimum wage might lead to a slack participation constraint—leave the agent a rent—the principal might want to add an NCC to the contract. This increases the equilibrium effort at no cost to the principal, merely the participation constraint gets tight—the agent's rent is reduced.

To uncover the welfare effects of an NCC, we decompose its effect into an idleness effect and an incentive effect. The *idleness effect* is the direct reduction in the social surplus from reducing the agent's payoff. The *incentive effect* works through the change in the equilibrium

[1] The fast-food firm *Jimmy John's* made its employees sign that they were not allowed to work for "any business which derives more than ten percent (10%) of its revenue from selling submarine, hero-type, deli-style, pita and/or wrapped or rolled sandwiches and which is located with three (3) miles of either [the Jimmy John's location in question] or any such other Jimmy John's Sandwich Shop." (Quoted from Jamieson (2014) in the *Huffington Post* `Jimmy John's Makes Low-Wage Workers Sign 'Oppressive' Noncompete Agreements`.) Jimmy John's has settled with the Attorney General in New York State and has stopped using non-compete clauses for sandwich workers in 2016. For more details, see Whitten (2016) for *CNBC* `Jimmy John's drops noncompete clauses following settlement`.

[2] On the federal level, the "Mobility and Opportunity for Vulnerable Employees Act", the "Workforce Mobility Act", and the "Freedom to Compete Act" have been introduced. Neither has been passed. Currently, the Federal Trade Commission considers banning NCCs because of anti-trust issues. There has been more progress on the state level: Some states now make NCCs unenforceable if the employee's salary lies below a threshold.

effort. We show that the incentive effect of NCCs transfers utility from the agent to the principal. If there is no bound on the NCC's severity, the profit maximizing contract makes the participation constraint bind and leaves the agent no rent, irrespective of the minimum wage's level. Our next result is concerned with the welfare effects of the utility transfer. Considering the utilitarian welfare,[3] the incentive effect is initially positive and becomes negative for large minimum wages: On the one hand, the incentive effect increases the equilibrium effort above the level without NCCs when it is inefficiently low (Proposition 3), on the other hand, the equilibrium effort with NCCs gets inefficiently large for high minimum wages (Proposition 4). The idleness effect always reduces the efficiency. If the minimum wage is low, introducing NCCs increases the utilitarian welfare as they reduce the inefficiency from minimum wages. If the minimum wage is large, introducing NCCs decreases the utilitarian welfare as the equilibrium effort get wastefully large.

In this model, minimum wages always reduce the utilitarian efficiency, however, a policy-maker that introduces minimum wages seems to care about the distribution of social surplus. Thus, we additionally compare equilibrium outcomes according to Pareto dominance. Both without and with bounded NCCs, the agent gets a rent if the minimum wage is sufficiently large. As soon as minimum wages become redistributive, in both cases, the utilitarian welfare remains constant in the minimum wage: Increasing the minimum wage by one unit increases the agent's payoff by one unit and decreases the principal's profit by one unit. Thus, whenever this constant utilitarian welfare is larger with a bounded NCC than with no NCC, strict Pareto improvements can be achieved by allowing bounded NCCs and suitably adjusting the minimum wage. An example verifies that bounded NCCs can indeed lead to a strict Pareto improvement over minimum wages alone.

Another, related contribution explains an empirical puzzle. Hair salon owners are more likely to make their employees sign NCCs when the minimum wage increases (Johnson and Lipsitz, 2020). Johnson and Lipsitz (2020) show that this can be explained if NCCs can be used to transfer utility. We complement their study by showing that effort provision is a possible micro-foundation for the utility transfer. Furthermore, the optimal contract in our model includes an NCC only if the minimum wage is sufficiently high and from thereon the optimal NCC gets strictly more severe in the minimum wage (Proposition 2).

Our second main contribution is to provide a novel explanation for why rational minimum wage workers sign NCCs in the first place. The provision of effort incentives complements the usual four reasons for the use of NCCs in low-wage jobs. Some of these reasons are not particularly appealing in the case of low-wage workers: Firstly, employers can use NCCs to improve their bargaining power in future wage bargaining.[4] Yet, minimum wage workers rarely bargain

[3]We define the utilitarian welfare as the unweighted sum of the of the agent's payoff and the principal's profit.

[4]The verbal argument is developed in Arnow-Richman (2006). Empirical findings from the ban of NCCs in the high-tech sector in Hawaii (Balasubramanian et al., 2020) are consistent with the argument. Moreover, 30% of the surveyed employees that have been bound by an NCC have not received any advanced notice, but have been presented their NCCs on their first day at work (Starr et al., 2019b, p. 21).

for wage increases.[5] Secondly, like non-disclosure agreements and non-solicitation agreements, NCCs protect proprietary information and client lists. Yet, many low-wage workers that do not have access to sensitive information are bound by NCCs (Starr et al., 2019b). Thirdly, NCCs increase the job tenure, which reduces the turnover.[6] This reduces training and hiring costs. Yet, these costs are rather low for most low-wage jobs.[7] Fourthly, NCCs mitigate the hold-up problem of investments in human capital. If workers are liquidity constrained and cannot invest in their industry-specific human capital, an NCC allows the employer to recoup her investment (Rubin and Shedd, 1981).[8] It is debatable how important human capital is in minimum wage jobs. One the one hand, it takes more than a thousand hours of training before one can become a certified hair dresser. On the other hand, many fast-food employees report that they have gotten less than three days of training. Nevertheless, our model does not rely on human capital or the other reasons to use NCCs. Instead, non-contractable effort is sufficient a reason to use NCCs.

Lastly, we contribute to the literature on moral hazard because we show that the equilibrium effort is non-monotone in the minimum wage (Proposition 3).

This paper is organized as follows. Section 2 provides background information on the use of non-compete clauses and their enforcement, and discusses the related literature. Section 3 introduces the model. In Section 4, we find the profit maximizing contracts in the benchmark, with unbounded NCCs, and with bounds on NCCs. The welfare implications of these contracts are analyzed in Section 5. In Section 6, we discuss the simplifying assumptions of our model and summarize empirical predictions of our model. Finally, Section 7 concludes.

2 Background and Related Literature

2.1 Background

As the legislation on non-compete clauses is very different across the United States, we focus on the aspects that are relevant for our model. The principal uses an NCC to threaten the agent into exerting more effort. For the threat to be credible, courts have to be willing to enforce such NCCs.

There are attempts by Bishara (2011) and Garmaise (2011) to compare whether the states' courts rule in favor of rather the employees or the employers. Both use a comprehensive

[5] Cahuc et al. (2006) find that low-wage worker possess no significant bargaining power. Instead, the wage growth for low-wage workers often comes from changing jobs, which is also shut down by NCCs (Colvin and Shierholz, 2019).

[6] A positive correlation of (the enforceability of) NCCs and the average length of job tenure has been found by Balasubramanian et al. (2020) and by Starr et al. (2019a).

[7] A meta-study (Boushey and Glynn, 2012) finds that the turnover costs average around 20% of the annual salary and are rather lower for low-skilled jobs. For the fast-food industry, reports range between 600$ and 2000$ while the turnover rate is around 150% (Rosenbaum, 2019). Yet, many firms do not even know their turnover cost and seem to ignore them as they are not salient (Altman, 2017).

[8] Long (2005) proposes repayment agreements as a better alternative to NCCs in this case. The disadvantage of NCCs is that they usually remain in the contract even after the employer has recouped his investment, whereas repayment agreements expire.

survey of courts' decisions (Malsberger, 2019) and questionnaires to calculate a one-dimensional measures of NCCs' enforceability for all states. This allows them to order the states on a spectrum, going from states that do not enforce NCCs at all—California, North Dakota, and Oklahoma—to states in which courts are ordered to ignore hardships that NCCs cause for employees—like Florida. In many states, employers can use NCCs in the way they want to. The differences across states can be captured in our model by different bounds on NCCs.

That NCCs might be used to provide incentives is also reflected in the enforceability questionnaire of Bishara (2011): "Question 8: If the employer terminates the employment relationship, is the covenant enforceable?" (Bishara, 2011, p. 777). The states are awarded scores on a scale from 0 to 10—where 0 means that a dismissal makes an NCC unenforceable and 10 means that a dismissal makes no difference whatsoever. Only five states score less than 6. Moreover, 15 jurisdictions score 10. That is, in most states having an NCC when being dismissed means worse opportunities on the job market.

There is further evidence that NCCs could be used to provide incentives. Often, courts do not even come into play. Employees often shy away from costly litigation even if their chances of winning are high. Some companies send their dismissed employees letters reminding them of their NCCs or request a preliminary injunction against a new employment. As a result, many former employees just sit their NCC out. Furthermore, some NCCs specify that trials are not held by courts but by mandatory arbitration. Since mandatory arbitrators' rulings are usually confidential, the enforceability of an NCC might differ from the expected enforceability in a given state.

There is also evidence that unenforceable NCCs are effective. Employees react to incredible threats, that is. Although California and North Dakota do not enforce NCCs, the prevalence is no different from that in states that enforce NCCs (Starr et al., 2019b). Moreover, these unenforceable NCCs have been shown to affect the employees' behavior (Starr et al., 2019c). This can be explained by the cost of litigation (Colvin and Shierholz, 2019, p. 5-6) or by irrational beliefs about the enforceability of NCCs. The availability bias might make employees overestimate the probability of enforcement due to the media reports on (few) dramatic cases. Employees might be uninformed about the laws in their state and, hence, fall for the bluff of their employer's attorney.

Summing up, there is a lot of evidence that suggests that NCCs can be used to incentivize employees. In many states, the laws allow this. Even if a states' law does not allow it, there are other ways to support the credibility of the threats. Lastly, even incredible threats seem to be somewhat effective.

2.2 Related Literature

This paper is related to multiple strands of literature. We will first summarize the small literatures on the incentive effects NCCs and on utility transfers using NCCs.[9] Then, we

[9] We refer the reader that is interested in other theoretical and empirical articles on NCCs to the survey McAdams (2019).

summarize two conceptually related concepts: efficiency wages and collateralized debt. Lastly, we contribute methodologically to the literature that uses similar moral hazard models.

In Kräkel and Sliwka (2009), contrasting our model, NCCs reduce the agent's incentives. In their model, exerting more effort increases the probability of outside offers. If the agent has no NCC, an outside offer leads to a wage increase to retain the agent. If the agent has an NCC, the principal does not have to make a retention offer; reducing the payoff from exerting effort.

Cici, Hendriock, and Kempf (2019) empirically test the incentive effect of NCCs. Their identification strategy is using exogenous legislative changes in the enforceability of NCCs. The hypotheses are derived without a formal model. They find that mutual fund managers perform better when NCCs get more enforceable. This evidence suggests the existence of the mechanism in our model.

NCCs have been argued before to redistribute rent from the agent to the principal. Wickelgren (2018) proposes a hold-up model with investments in human capital. A minimum wage prevents the principal from extracting all rents without an NCC. By making the agent sign an NCC, the principal can prevent the agent from leaving without increasing the wage. The optimal contract does not leave a rent to the agent. In contrast to our work, this model relies on human capital investments for minimum wage workers.

Johnson and Lipsitz (2020) find in the data that higher minimum wages lead to more NCCs. They also provide a model on the use of NCCs to transfer utility if a minimum wage restricts the transfer of utility via money. If the terms of trade favor the employers, the employees have to sign NCCs to (inefficiently) transfer utility to the employers in equilibrium. When signing an NCC, employees incur an exogenous cost while employers receive an exogenous benefit. Whether NCCs are used or not is determined by the participation constraint of the least productive firm according to a "law of one price." Thus, the prediction that larger minimum wages lead to the use of NCCs. We complement their work by providing a micro-foundation for NCCs' transferring utility.

The interpretation of non-compete clauses as a means to provide incentives reminds of two similar concepts. Firstly, there are efficiency wages. In the literature started by Shapiro and Stiglitz (1984), an agent's retaining a job gets him a rent relative to the outside option. Failing to retain a job because of a bad performance means receiving the outside option. The difference in payoffs between being retained or not makes the agent exert effort. In our model, this is the source of the agent's incentives, too. The difference is that the agent's payoff, if he is not retained, is below the outside option; it is being unemployed for the duration of the NCC. If the agent is retained, his payoff is above the outside option. Moreover, with unbounded NCCs, the agent's expected payoff always is equal to the outside option. That is, the agent gets no rent from signing the contract.

Secondly, there is the literature on collateralized debt (e.g. Stiglitz and Weiss, 1981, Bernanke and Gertler, 1986, Chan and Thakor, 1987, Bester, 1987, Boot et al., 1991, and Tirole, 2006). An agent, who is cash constrained, might pledge an asset in order to improve his access to a credit line. After a signal for low effort (default), the asset is transferred to the bank.

This both incentivizes the agent and reduces the loss of the bank. Non-compete clauses in our model are similar to collaterals in lending agreements: The agent pledges his human capital. After a bad performance, the NCC is activated, and the agent is not allowed to sell his human capital to someone else. One difference in these articles is the efficiency loss from using other payoff dimensions. For example, pledging a perfectly resellable asset is a perfect substitute to monetary payments. Thus, the friction from limited liability vanishes. In the other extreme, the principal inflicts costly pain on the agent (Chwe, 1990), causing inefficiency twice. Still, the existence of the second argument in the agent's payoff makes the principal better off. Our model is in-between these extremes: An NCC is costly to the agent but not to the principal.

Methodologically, we contribute to the literature of agency models with moral hazard in continuous effort and with limited liability (e.g. Schmitz, 2005, Kräkel and Schöttner, 2010, Ohlendorf and Schmitz, 2012, and Englmaier et al., 2014). Especially, we contribute to the agency literature with multidimensional (monetary and non-monetary) payoffs. In our model, the payoff's dimensions are present and future payoff. Minimum wages affect only present payoffs. NCCs can reduce only future payoffs via unemployment. As in the present paper, Kräkel and Schöttner (2010) show that future rents can be used to incentivize effort in the first period.

There are articles with similar models that interpret the second argument of the agent's payoffs as pain or unfriendliness. It is pain in the coerced labor settings of Chwe (1990) and Acemoglu and Wolitzky (2011). Chwe (1990) provides a model in which the principal can inflict costly pain to the agent. As in our model, inflicting pain maximizes the profit if monetary transfers are limited due to wealth constraints. Another variant of this model is used in Acemoglu and Wolitzky (2011). Besides some simplifications, the principal can pay to reduce the agent's reservation utility. Furthermore, the model is later extended from a partial market to a general equilibrium. The interpretation as unfriendliness is taken in Dur et al. (2019). Under limited liability a leader might use an unfriendly leadership style to reduce the agent's payoff.

3 The Model

We consider a moral hazard model with continuous effort, binary output, and limited liability. There is a risk-neutral principal P (*she*) who owns a project. The project can be either a success and pay off V or a failure and pay off nothing. P wants to hire a risk-neutral agent A (*he*) to work on the project for one period. The principal offers the agent a contract that consists of three items: a base wage w, which is paid unconditionally, a bonus wage b, which is paid conditionally on a success, and a non-compete clause (NCC). The wages are subject to a minimum wage that introduces limited liability. Our model consists of two parts: the effort provision stage and a continuation in which an NCC might come into play.

We now consider the effort provision stage in more detail. The agent chooses his effort $e \in [0, 1]$ at a strictly convex cost of $c(e)$, where $c(0) = 0$. We assume the standard Inada conditions that $c'(0) = 0$ and $\lim_{e \to 1} c'(e) = \infty$. We also assume that $\frac{c'''(e)}{c''(e)} > \frac{1}{1-e} \quad \forall e \in [0, 1)$ to get a concave objective function (see Lemma 7 in the Appendix).[10] Two examples are $c(e) = -\ln(1-e) - e$ and $c(e) = \frac{e^2}{1-e}$.[11] The effort level that A chooses is private information and, thus, creates a moral hazard problem. The chosen effort is the probability of the project being successful, that is, a success payoff V accrues to the principal with probability e, $\text{Prob}(\text{success}|\ e) = e$. Successes are verifiable and serve as a signal for the agent's effort. In the case of a success, the agent gets the bonus wage b.

We now consider the continuation in more detail. After the project is completed, the agent's continuation payoff is determined. For simplicity, we do not model future periods, but define the discounted future life-time payoff as a function of the project's outcome and of the non-compete clause. We assume that there is no other moral hazard problem (or any other friction) in the future such that the agent will never receive a rent. Therefore, the continuation payoff can take two different values. Firstly, if the project is a success, the agent is retained. Since there is no dismissal, an NCC does not make any difference. As the agent never earns a rent in the future, the continuation payoff is the outside option, which we normalize to zero. Secondly, if the project ends in a failure, the agent is dismissed. In this case, the continuation payoff depends on whether the agent is bound by a non-compete clause. If the agent's contract does not entail an NCC, he is free to pursue his outside option, so his continuation payoff is zero; the same as if he had been retained. If the agent has signed an NCC, he will be unemployed for a while before he is allowed to pursue his outside option. The unemployment reduces his continuation payoff to $\bar{v} \leq 0$. For simplicity, we express the contract's NCC directly as the reduction in the agent's continuation payoff, $\bar{v}$. We discuss this assumption in Section 6. Concerning the principal, we assume that dismissing the agent has no effect on her continuation payoff. That is, hiring a replacement is costless.

To sum up, a contract between the principal and the agent is defined by the tuple $(w, b, \bar{v})$. These items are constrained. The minimum wage law demands that the agent is paid at least the minimum wage $\underline{w}$ for the effort-provision stage.[12] After a failure, the principal pays the agent $w \geq \underline{w}$, and after a success she pays him $w + b \geq \underline{w}$. The level of the minimum wage is relative to the agent's outside option that we have normalized to zero.[13] The NCC is constrained

[10]Compared to the canonical principal agent model, the principal has an additional choice variable, the NCC. Therefore, to get a well-behaved problem, we need a stronger assumption on the cost function than the standard assumption that $c'''(e) > 0$. Chwe (1990) and Acemoglu and Wolitzky (2011) use the same assumption in their models. In the proofs in the Appendix, we will state which assumptions on the cost function we need in the respective steps. The concavity assumption is simpler and implies all of them.

[11]Note that these cost functions are only defined for $e \in [0, 1)$. Therefore, we consider $c(e) = \infty$ for $e = 1$.

[12]A minimum wage might not be the only friction that prompts employers to use NCCs. In general, employers want to use NCCs as soon as employee's receive some kind of rent that can be expropriated via NCCs. For example, the downward-rigidity of nominal wages might prevent the principal from reducing the wages but not from adding an NCC. Cici et al. (2019) have shown that NCCs have incentive effects on funds managers, which suggests that funds managers receive some rent. An example for a rent that cannot be expropriated using NCCs is an information rent: Anticipating that he would be asked to sign an NCC, an employee will not reveal his private information.

[13]In other models with limited liability, it is often assumed that the agents are heterogeneous in their outside options. Then, the minimum wage (or limited liability) is normalized to zero. In our model, on the contrary, we normalize the outside option to zero. This reflects our assumption that ability or human capital plays no role, thus, the agents are homogeneous in their outside options. We are interested in the effects of (an increase

because it can only reduce the agent's continuation payoff, $\bar{v} \leq 0$. We say that a contract does have *no non-compete clause* if $\bar{v} = 0$. The lower $\bar{v}$, the longer the agent is unemployed after being dismissed. We refer to a lower $\bar{v}$ as a *more severe non-compete clause.*

The agent' expected utility is given by

$$\mathbb{E}U = w + e \cdot b + (1 - e) \cdot \bar{v} - c(e). \tag{1}$$

The agent takes his contract as given and chooses his effort. The base wage is paid unconditionally, the bonus wage only in the case of success and the NCC is activated only in the case of failure.

The principal's expected profit is given by

$$\pi = -w + e \cdot (V - b). \tag{2}$$

The principal anticipates the agent's effort decision and chooses the contract to offer to the agent. Up to Section 5, we assume that the success payoff, V, is large enough such that the principal makes a profit that exceeds her outside option, and we ignore the extensive margin.

The timing of the game is as follows. The principal offers a contract to the agent. The agent can reject or accept the offer. If he rejects, the game ends and he gets his outside option. If he accepts, the game continues. The agent then chooses his effort from the unit interval. The payoffs to the agent and the principal are determined according to the accepted contract, including that the principal dismisses the agent and activates the NCC after a failure. The solution concept is the subgame perfect Nash equilibrium. We find it by backward induction. The timeline in Figure 1 summarizes the game.

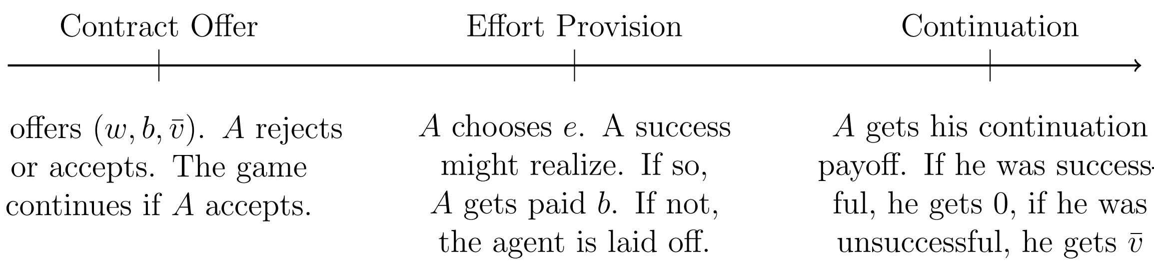

Figure 1: Timing of the Game.

First-Best Welfare Analysis. First, consider the benchmark without any frictions. A social planner maximizes the expected social welfare

$$W^{FB} = \max_{e \in [0,1],\ \bar{v} \leq 0} \ e \cdot V - c(e) + (1 - e) \cdot \bar{v}. \tag{3}$$

in) the minimum wage. Our modeling choice makes this more easily interpretable. To consider heterogeneous agents, keep in mind that a better outside option is equivalent to a lower minimum wage.

The first-order condition shows that in the social optimum there is no NCC because due to the Inada conditions, the effort will be interior. As a result, any NCC comes into action with positive probability and forces the agent's labor to lie idle. This is inefficient. This means that $\bar{v} = 0$ maximizes the social surplus.

Given that $\bar{v} = 0$, the first-best effort equates the marginal benefit and the marginal cost, $V = c'(e^{FB})$. This is optimal due to the welfare function's concavity.

4 The Optimal Contract

In this section, we characterize the profit maximizing contracts for different minimum wages. We start with analyzing how the presence of an NCC changes the incentive and the participation constraint and reformulate the participation constraint to see how an NCC affects the model. We then solve for the optimal contract in the benchmark, in which the principal is not allowed to use NCCs. Then, we allow the principal to use arbitrarily severe NCCs, and we compare the results to the benchmark. We find that the principal only wants to use NCCs if the minimum wage is sufficiently large. Moreover, there is no minimum wage for which the agent gets a rent. Lastly, we limit the severity of NCCs, as it is in reality. Formally, we introduce a lower bound, such that $\underline{v} \leq \bar{v} \leq 0$. The bound limits the principal's power to extract the agent's rent: From some minimum wage on, the agent is left a rent.

4.1 The Equilibrium Analysis

To build intuition, we now look at how an NCC changes the incentive compatibility and the participation constraint. Given the contract $(w, b, \bar{v})$, the agent chooses the effort level e^* that maximizes his expected utility,

$$e^* = \underset{e \in [0,1]}{\arg\max} \; w + e \cdot b + (1 - e) \cdot \bar{v} - c(e). \tag{IC}$$

This is the agent's incentive compatibility constraint. If $b - \bar{v}$ is non-negative, the agent's effort choice is characterized by the first-order condition

$$b - \bar{v} = c'(e^*). \tag{4}$$

The equilibrium effort is then unique because the marginal cost is strictly increasing. The first-order condition shows that the bonus wage and the NCC are perfect substitutes for giving incentives. Therefore, NCCs have an *incentive effect* as they generate higher effort incentives. P must now decide to what extent to provide incentives through an NCC and to what extent through a bonus wage.

The agent only accepts the contract if his participation constraint

$$w + e^* \cdot b + (1 - e^*) \cdot \bar{v} - c(e^*) \geq 0. \tag{PC}$$

is satisfied. The bonus wage and the severity of the NCC go into opposite directions in the participation constraint. A higher bonus wage makes the participation constraint slack.[14] A more severe NCC makes the participation constraint tight. This already hints at the use and the distributional effects of NCCs: Whenever the agent would get a rent without an NCC, the principal will add an NCC to the contract and convert the rent into more incentives. The participation constraint will always bind. In the participation constraint, the NCC enters twice. Firstly, it enters indirectly via the equilibrium effort, through the incentive effect. Secondly, the idleness effect can be seen in the participation constraint: The NCC enters directly as $(1-e^*)\bar{v} \leq 0$. That is, the NCC reduces the agent's utility (and, thus, the social surplus) because labor force has to lie idle if there is a failure.

One could also decompose the effect of an NCC in a different way. Rearranging the agent's expected utility yields $(w+\bar{v})+e^*(b-\bar{v})-c(e^*)$. This means that the NCC effectively reduces the base and increases the bonus wage. Thus, the NCC allows the principal to effectively circumvent the minimum wage law to some extent. The reduction in the base wage, however, does not benefit the principal because the NCC does not enter the principal's profit directly. Instead, the principal benefits from the increase in the (effective) bonus wage, as the difference between a failure and a success gets larger for the agent. In this reformulation, the incentive effect is hidden in the equilibrium effort and the idleness effect is the two $\bar{v}$ in the expected utility. In Section 5, we will take a closer look at the welfare effects of the incentive and the idleness effect.

With the possibility of imposing an NCC, the principal's problem becomes

$$\max_{w,b,\bar{v}} \quad -w + e^* \cdot (V-b) \tag{5}$$

$$\text{subject to} \qquad e^* = \arg\max_{e\in[0,1]} \; w + e \cdot b + (1-e)\cdot \bar{v} - c(e) \tag{IC}$$

$$w + e^* \cdot b + (1-e^*)\cdot \bar{v} - c(e^*) \geq 0 \tag{PC}$$

$$\bar{v} \leq 0 \tag{NCC}$$

$$w \geq \underline{w} \qquad w + b \geq \underline{w} \qquad \text{(MWC1) and (MWC2)}$$

The principal maximizes her expected profit subject to the incentive-compatibility constraint, the participation constraint, the NCC feasibility constraint and the minimum wage constraints.

The Benchmark Without Non-Compete Clauses. Before we proceed and analyze the optimal contract with NCCs, we briefly consider the benchmark without NCCs. Formally, this means that P cannot freely choose $\bar{v}$ but that $\bar{v} \overset{!}{=} 0$ exogenously. Hence, P can only choose the base and the bonus wage. The optimal contracts under limited liability with those two tools are well known (see for example Laffont and Martimort, 2002, and Schmitz, 2005). Proposition 1 derives the optimal contract that the principal offers to the agent in the benchmark.

[14]That is the reason why limited liability usually causes the agent to get a rent: To provide incentives, the principal has to pay a bonus wage, slackening the participation constraint.

Proposition 1. *Consider the problem without NCCs. There exist threshold values in the minimum wage κ_1 and κ_3 such that P offers the following contract to A:*

(i) Let $\underline{w} \leq \kappa_1$. Then P chooses the compensation scheme $(w, b) = (\kappa_1, V)$.

(ii) Let $\kappa_1 < \underline{w} \leq \kappa_3$. Let $e_2^{BM}(\underline{w})$ be implicitly defined by $c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$. Then P chooses the compensation scheme $(w, b) = (\underline{w}, c'(e_2^{BM}))$.

(iii) Let $\kappa_3 < \underline{w}$. Let $e_3^{BM}(\underline{w})$ be implicitly defined by $c'(e_3^{BM}) + e_3^{BM} \cdot c''(e_3^{BM}) = V$. Then P chooses the compensation scheme $(w, b) = (\underline{w}, c'(e_3^{BM}))$.

Proof. See Appendix A.1. □ □

Note that κ_2 that lies in-between κ_1 and κ_3 plays a role only with NCCs but not in the benchmark.

The three parts of Proposition 1 correspond to the three cases of binding and non-binding constraints; depending on the level of the minimum wage.

Case 1. The minimum wage is lower than the wages the principal wants to sets the base wage when she ignores the minimum wage constraints. Therefore, the optimal contract is the same as with unlimited liability. The principal leaves the success payoff to the agent and uses the base wage to extract the complete surplus from the agent. Therefore, this case is commonly referred to as "selling the firm."

Case 2. If the minimum wage is above κ_1, selling the firm violates the minimum wage condition; the principal cannot extract the full social surplus anymore. The optimal base wage is the minimum wage. It is better for the principal to keep the agent's participation constraint satisfied with the bonus wage than with the base wage, as this provides incentives. For minimum wages between κ_1 and κ_3, the bonus wage that solves the principal's first-order condition of profit maximization would violate the participation constraint; it is too low. Because the profit function is concave in the bonus wage, the optimal bonus wage makes the participation constraint binding. It is below the success payoff, implying a lower than first-best effort and social surplus. Furthermore, it is decreasing in the minimum wage as a lower bonus wage is needed to keep the participation constraint satisfied when the base wage is larger. Hence, the equilibrium effort is also decreasing in the minimum wage. The binding participation constraint means that the minimum wage does not redistribute from the principal to the agent; it solely induces inefficiency.

Case 3. For minimum wages above κ_3, the bonus wage that solves the principal's first-order condition does not violate the participation constraint anymore; as the base wage is large enough. As the first-order condition does not depend on the minimum wage, neither does the optimal bonus wage; and thus nor does the equilibrium effort. The social surplus is, thus, constant. Because the participation constraint does not bind anymore, the agent gets a rent.

A minimum wage now becomes a tool of perfect redistribution: An increase of the minimum wage by one unit translates into an increase of the agent's rent by one unit.

The Equilibrium Analysis With Non-Compete Clauses. Proposition 2 summarizes the optimal contracts with NCCs.

Proposition 2. *Consider the problem with NCCs. There exist threshold values in the minimum wage κ_1 and κ_2. If $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V < 1$, there exist the threshold value in the minimum wage κ_4. P offers the following contract to A:*

(i) Let $\underline{w} < \kappa_1$. Then P offers the contract $(w, b, \bar{v}) = (\kappa_1, V, 0)$.

(ii) Let $\kappa_1 \leq \underline{w} \leq \kappa_2$. Let $e_2^{BM}(\underline{w})$ be implicitly defined by $c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$. Then P offers the contract $(w, b, \bar{v}) = (\underline{w}, c'(e_2^{BM}), 0)$.

(iii) Let $\kappa_2 < \underline{w} < \kappa_4$. Let $e_3^{NCC}(\underline{w})$ be implicitly defined by $c(e_3^{NCC}) + (1 - e_3^{NCC})c'(e_3^{NCC}) + e_3^{NCC}(1 - e_3^{NCC})c''(e_3^{NCC}) = V + \underline{w}$. Then P offers the contract $(w, b, \bar{v})$ $= \left(\underline{w}, (1 - e_3^{NCC})c'(e_3^{NCC}) + c(e_3^{NCC}) - \underline{w}, c(e_3^{NCC}) - \underline{w} - e_3^{NCC}c'(e_3^{NCC})\right)$.

(iv) Let $\kappa_4 \leq \underline{w}$. Let $e_4^{NCC}(\underline{w})$ be implicitly defined by $(1 - e_4^{NCC}) \cdot c'(e_4^{NCC}) + c(e_4^{NCC}) = \underline{w}$. Then P offers the contract $(w, b, \bar{v}) = \left(\underline{w}, 0, -\frac{\underline{w} - c(e_4^{NCC})}{1 - e_4^{NCC}}\right)$.

Proof. See Appendix A.2. □

The four parts of Proposition 2 correspond the the four combinations of binding and non-binding constraints for different levels of the minimum wage. Figure 2 illustrates which constraints are binding in the optimum depending on the size of the minimum wage. If $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V \geq 1$, Combination 4 is never optimal. Importantly, the participation constraint binds in all combinations; the agent never gets a rent. If the participation constraint were slack, there would be a profitable deviation: making the NCC more severe. The equilibrium effort increases, and because the agent gets less than the success payoff, the principal profits.

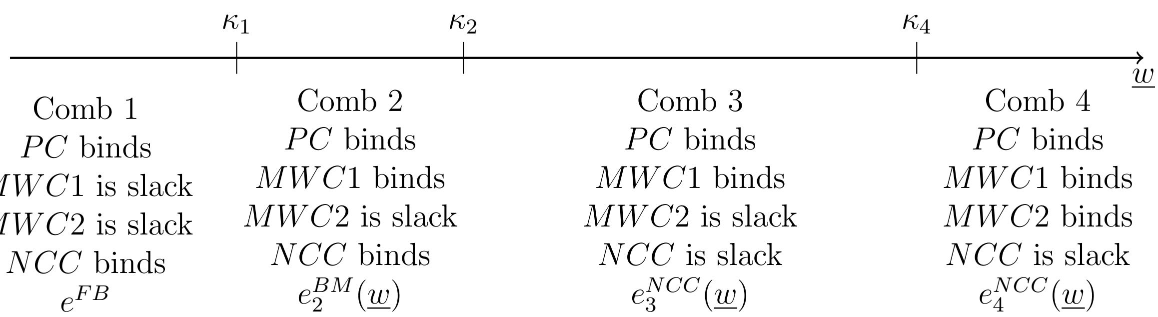

Figure 2: The Combinations of Binding and Non-Binding Constraints that Characterize the Optimal Contract when NCCs are Allowed. The Combinations are from Table 1. κ_3 does not have any meaning in the world with NCCs.

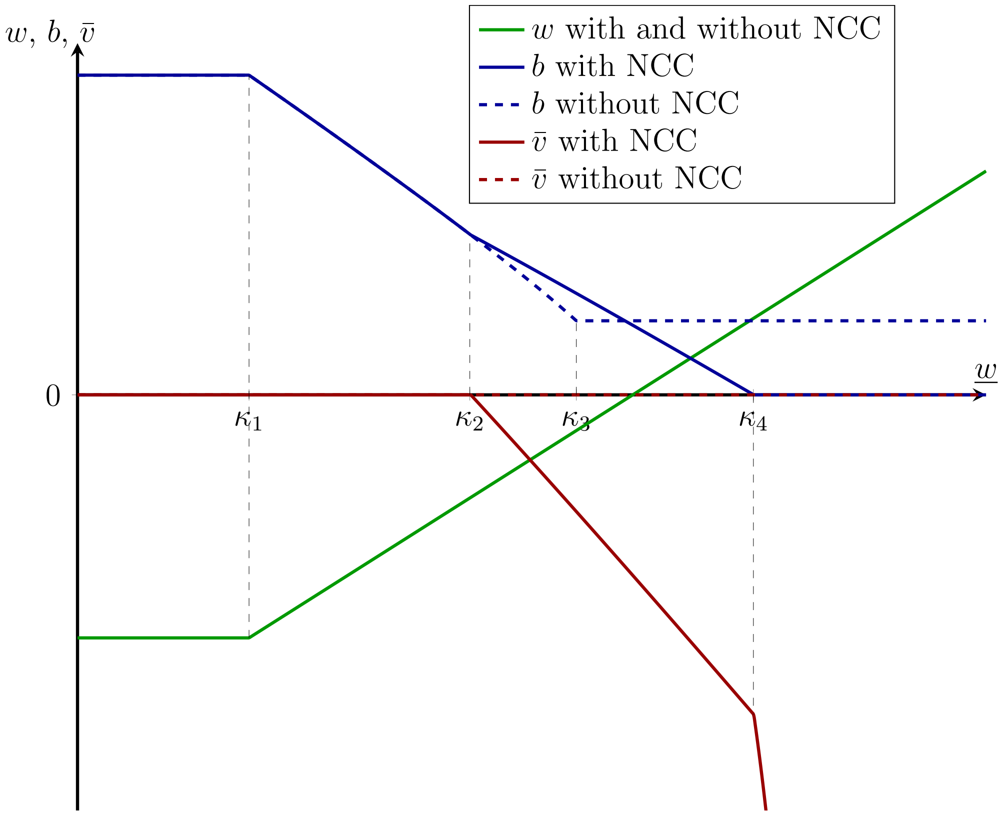

Figure 3: Illustration of the Optimal Contract for Different Minimum Wages when P can use NCCs and when she cannot use them for $c(e) = -\ln(1-e) - e$ and $V = 10$.

Another illustration of the optimal contract for a parameterized cost function is given in Figure 3. This figures compares the optimal contract when the principal can use NCCs and when she cannot.

We will now consider each combination in more detail.

Combination 1. This combination is identical to Case 1 in the benchmark. As the principal's profit is already equal to the first-best social surplus, she cannot do any better by introducing an NCC.

Combination 2. For minimum wages between κ_1 and κ_2, it is optimal for the principal not to use NCCs. The optimal contract is the same as in the benchmark in Case 2, although it stops at a lower minimum wage, $\kappa_2 < \kappa_3$. Because the equilibrium effort is still quite high at these minimum wages and because the effort cost is convex, additional incentives increase the effort only by little. Therefore, the incentive effect of an NCC is small. On the other hand, there is the idleness effect: As the participation constraint binds, the principal would have to increase the bonus wage to compensate the agent for NCC's effect after a failure. The idleness effect is larger. Considering the small incentive effect, using an NCC is too costly.

Combination 3. As the minimum wage increases, the bonus wage and, hence, the equilibrium effort decrease. At a lower equilibrium effort level, again because of the convex effort cost, the incentive effect of an NCC is larger. Simultaneously, the idleness effect gets linearly larger in the increasing probability of a failure. Due to the convex effort cost, the incentive effect changes more. At a minimum wage of κ_2, both effects are equally strong. If the minimum wage is above κ_2, the incentive effect prevails and the principal uses an NCC. Moreover, a side product of Proposition 3 shows that the optimal NCC gets monotonically more severe in the minimum wage.

Not only does the optimal NCC get more severe in the minimum wage, but also does the sum of incentives increase in the minimum wage (Proposition 3 (iii)). That is, the change in the NCC overcompensates the decreasing bonus wage. Thus, the equilibrium effort is strictly increasing for minimum wages above κ_2 (as we show below, the equilibrium effort remains increasing even if Combination 4 is optimal). As argued above, our modelling assumptions lead to the existence of a range of minimum wages in which Combination 2 is optimal. In this range, the equilibrium effort is strictly decreasing in the minimum wage. Combining these findings yields a novel result: The equilibrium effort is non-monotone in the minimum wage (Proposition 3). Standard models in the literature (like our benchmark) find that the equilibrium effort is decreasing and finally stays constant in the minimum wage.

Proposition 3 (Non-Monotonicity of Optimal Effort)**.** *The equilibrium effort is non-monotonic in the minimum wage.*

(i) Let $\underline{w} < \kappa_1$. Then the equilibrium effort level is constant in the minimum wage.

(ii) Let $\kappa_1 \leq \underline{w} \leq \kappa_2$. Then the equilibrium effort level is strictly decreasing in the minimum wage.

(iii) Let $\kappa_2 < \underline{w}$. Then the equilibrium effort level is strictly increasing in the minimum wage.

Proof. See Appendix A.3. □

The equilibrium effort is not only increasing in the minimum wage, but it also gets eventually inefficiently large, as Proposition 4 shows. Using more severe NCCs is the principal's only way of extracting the agent's rent. The principal accepts inducing inefficiently large efforts as some of the social surplus would otherwise end up with the agent. If the minimum wage goes to infinity, the equilibrium effort goes to one.

The equilibrium effort with and without NCC is illustrated in Figure 8 in Appendix B for a parameterized cost function.

Proposition 4 (Inefficiently Large Optimal Effort)**.** *The equilibrium effort exceeds the first-best effort if the minimum wage is sufficiently large.*

Proof. See Appendix A.4. □

How does Combination 3 compare to Case 2 in the benchmark? As the optimal contract in Case 2 makes the participation constraint bind without an NCC, the bonus wage in Combination 3 has to be larger. With the possibility to use an NCC, the principal can provide "double incentives" by increasing the bonus wage: Higher bonus wages make the participation constraint slack. This allows for a more severe NCC, which makes the participation constraint binding again. Both the increase in the bonus wage and in the NCC's severity provide incentives. In the proof of Proposition 3, we also show that the optimal bonus wage nevertheless decreases in the minimum wage, although less so than in Case 2 of the benchmark. At some minimum wage above κ_3, the optimal bonus wage with NCCs falls below the optimal, constant bonus wage from Case 3. Intuitively, the minimum wage is so large that it keeps the participation constraint satisfied even with a smaller bonus wage. In Case 3, the bonus wage is so large that it makes the participation constraint slack and the agent gets a rent.

Combination 4. The effectiveness of the double incentives decides whether Combination 3 remains optimal for all minimum wages: Using a bonus wage for all minimum wages is only optimal, if the equilibrium effort reacts strongly enough to an increase in the incentives. The formal condition on the effort cost function is $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V \geq 1$. If this condition is not satisfied, at a minimum wage of κ_4, it becomes profitable for the principal to not pay a bonus wage anymore; Combination 4 becomes optimal. It is then better to provide incentives only through the NCC instead of using double incentives.[15] The saved cost outweighs the reduction in the equilibrium effort, as the equilibrium effort does not react much.

For the same reason, in contrast to Combination 3, the principal would even want to pay negative bonus wages—charging the agent for successes—as the equilibrium effort gets inefficiently large. This would be another way to extract the agent's rent than via more effort. The minimum wage condition for the case of a success, however, would be violated if the base wage is not adjusted: To use a negative bonus wage, the base wage has to be above the minimum wage. As we show in the proof of Proposition 2, increasing the base wage is too expensive for the principal. Therefore, the optimal bonus wage can never be negative.

Figure 4 shows an overview of the optimal contract that P offers to A in the cases in which NCCs are prohibited and allowed. This Figure mainly summarizes the previous results.

[15] This might be one answer to the empirical question why we do not see that many bonus wages in reality; even if it is possible to condition payments on performance measures. Other forms of (implicit) incentives reduce the marginal benefit of using bonus wages.

NO NCC

$\kappa_1 = c(e^{FB}) - e^{FB}c'(e^{FB})$

$\kappa_3 = c(e_3^{BM}) - e_3^{BM}c'(e_3^{BM})$

Case 1

$$w = \kappa_1$$
$$b = V$$
$$e = e^{FB}$$

Case 2

$$w = \underline{w}$$
$$b = c'(e_2^{BM})$$
$$e_2^{BM}(\underline{w}) \ : \ c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$$

Case 3

$$w = \underline{w}$$
$$b = c'(e_3^{BM})$$
$$e_3^{BM} \ : \ c'(e_3^{BM}) + e_3^{BM} \cdot c''(e_3^{BM}) = V$$

$\underline{w}$

Combination 1

$$w = \kappa_1$$
$$b = V$$
$$\bar{v} = 0$$
$$e = e^{FB}$$

Combination 2

$$w = \underline{w}$$
$$b = c'(e_2^{BM})$$
$$\bar{v} = 0$$
$$e_2^{BM}(\underline{w}) \ : \ c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$$

Combination 3

$$w = \underline{w}$$
$$b = (1 - e_3^{NCC})c'(e_3^{NCC}) + c(e_3^{NCC}) - \underline{w}$$
$$\bar{v} = c(e_3^{NCC}) - \underline{w} - e_3^{NCC}c'(e_3^{NCC})$$
$$e_3^{NCC}(\underline{w}) \ : \ c(e_3^{NCC}) + (1 - e_3^{NCC})c'(e_3^{NCC}) + e_3^{NCC}(1 - e_3^{NCC})c''(e_3^{NCC}) = V + \underline{w}$$

Combination 4

$$w = \underline{w}$$
$$b = 0$$
$$\bar{v} = -\frac{\underline{w} - c(e_4^{NCC})}{1 - e_4^{NCC}}$$
$$e_4^{NCC}(\underline{w}) \ : \ \underline{w} + e_4^{NCC}c'(e_4^{NCC}) - c(e_4^{NCC}) = c'(e_4^{NCC})$$

$\kappa_1 = c(e^{FB}) - e^{FB}c'(e^{FB})$

$\kappa_2 : \left.\frac{\partial \pi}{\partial b}\right|_{b=b_2^{**}} = 0$

κ_4: $\left.\frac{\partial \pi}{\partial b}\right|_{b=0} = 0$ (if it exists)

NCC

Figure 4: The Profit-Maximizing Contracts for Different Minimum Wages Without (Above) and With (Below) NCCs.

4.2 Bounded Non-Compete Clauses

Having characterized the optimal contracts with unbounded non-compete clauses, we now extend to bounded non-compete clauses. We define $\underline{\bar{v}} < 0$ as the most severe NCC that the principal may use. The additional constraint takes the form $\bar{v} \geq \underline{\bar{v}}$.

We formalize our findings as Proposition 5.

Proposition 5 (Bounded Non-Compete Clauses)**.** *Let $\underline{\bar{v}} < 0$ be a lower bound on the NCC.*

(i) Let, without a bound on NCCs, the optimal NCC be $\bar{v} \geq \underline{\bar{v}}$. Then, the optimal contract remains the same with a bound on NCCs.

(ii) Let, without a bound on NCCs, the optimal NCC be $\bar{v} < \underline{\bar{v}}$. Then, the optimal contract with a bound on NCCs has $\bar{v} = \underline{\bar{v}}$. If the optimal bonus wage is positive, when the bound on the NCC starts binding, the bonus wage decreases more steeply than without a bound. At some larger minimum wage, the optimal bonus wage becomes constant either at a positive level or at zero. If the optimal bonus wage is zero, when the bound on the NCC starts binding, the bonus wage remains at zero for all larger minimum wages.

Proof. See Appendix A.5. □

Some supplementary remarks: As the profit-maximizing NCC gets infinitely severe if the minimum wage goes to infinity, it will eventually reach the bound.

After the bound is reached, positive bonus wages decrease faster than without a bound because there are no more double incentives. Increasing the bonus wage means that the agent gets a rent that cannot be converted into more incentives as the NCC cannot be made more severe. This reduces the benefit of bonus wages.

As soon as the NCC has reached the bound and the bonus wage remains constant, there is redistribution as in the benchmark. The minimum wage at which the bonus wage becomes constant is larger than that in the benchmark, κ_3. We will consider this in more detail in the next section.

For illustration, Figure 5 shows the optimal contract with bounded NCCs for a specific effort cost function and a specific bound. In the depicted case, the optimal constant bonus wage is positive. The optimal contract is the same as without a bound up to a minimum wage slightly above κ_2. Then, the bound on the NCC starts to bind and the optimal bonus wage has a kink. Somewhere to the right of κ_3, the participation constraint gets slack and the optimal bonus wage gets constant. If the bound on the NCC were looser, the optimal constant bonus wage might be zero.

5 Welfare Analysis

5.1 Utilitarian Welfare

Having characterized the profit-maximizing contracts, we can now look at the welfare effects of NCCs. It is interesting to look at the utilitarian welfare—the sum of the agent's rent and

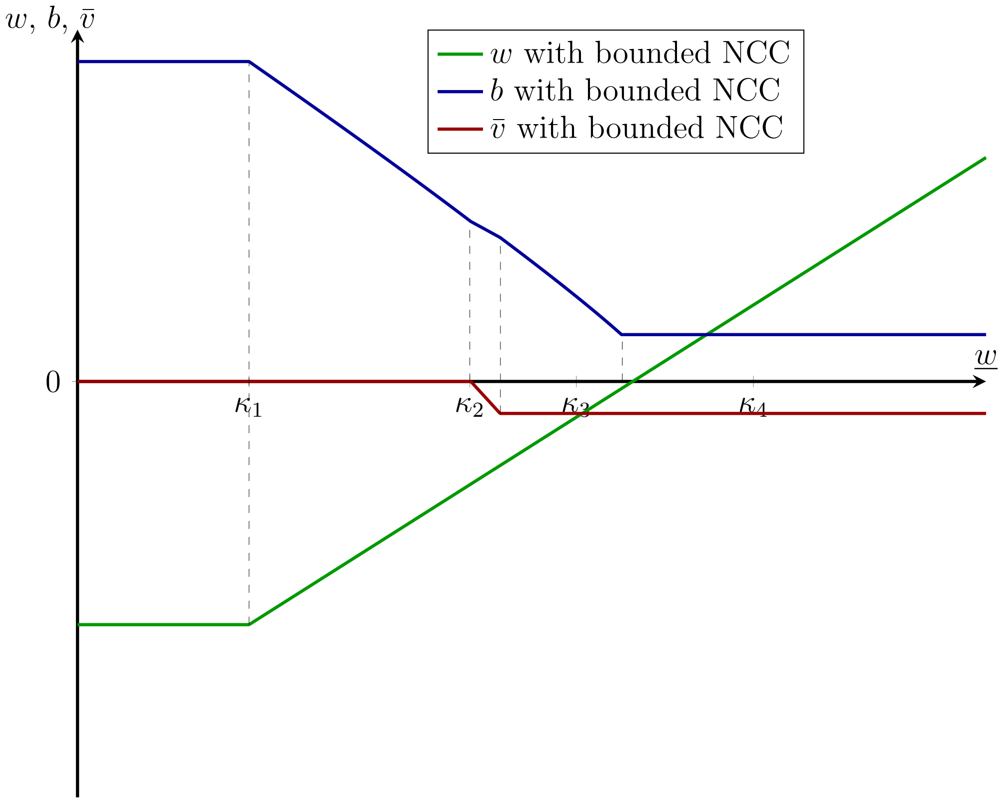

Figure 5: Illustration of the Optimal Contract for Different Minimum Wages for $c(e) = -\ln(1-e) - e$, $V = 10$ and a Bound on the NCC of $\underline{v} = -1$.

the principal's profit—in the case of unbounded NCCs. From the previous section, we already know that if NCCs are unbounded, the agent never gets a rent. Hence, the utilitarian welfare is equal to the principal's profit.

NCCs affect the utilitarian efficiency through two channels: the incentive and the idleness effect. The incentive effect works indirectly through the increasing equilibrium effort due to an NCC. The incentive effect formally is

$$\int_{e^{\text{No NCC}}}^{e^{NCC}} (V - c'(x)) \; dx, \tag{6}$$

where $e^{\text{No NCC}}$ denotes the equilibrium effort without an NCC and e^{NCC} denotes the equilibrium effort with an NCC, both of which depend on the minimum wage. The incentive effect is zero at the minimum wage κ_2, at which the principal first uses an NCC. It then increases in the minimum wage and finally decreases again: For minimum wages slightly above κ_2, without an NCC, the equilibrium effort is inefficiently low. An NCC moves the equilibrium effort closer to the first-best; the incentive effect is positive. As long as the equilibrium effort with an NCC lies below the first-best, the incentive effect is certainly positive. For large minimum wages, however, the equilibrium effort gets wastefully large as it increases above the first-best level (Proposition 4). At an even larger minimum wage, thus, the incentive effect starts to decrease. Finally, from some minimum wage on, the equilibrium effort is large enough to make the incentive effect negative.

The idleness effect directly affects the utilitarian inefficiency by reducing the agent's payoff. In the case of a failure, the NCC gets activated and burns $\bar{v}$ of the social surplus. Thus, this effect is unambiguously negative. It formally is

$$(1 - e^{\text{NCC}}) \cdot \bar{v}, \tag{7}$$

where e^{NCC} again denotes the equilibrium effort with an NCC, that depends on the minimum wage. The NCC's total effect on the utilitarian efficiency is the sum of the two effects.

We split the comparison with the benchmark into two parts: for those minimum wages below κ_3, and those above κ_3 because of properties of the benchmark: Without an NCC, the agent gets no rent for minimum wages below κ_3. The principal's profit is also equal to the utilitarian welfare, which makes the comparison simple. Whenever the principal uses an NCC, she does so because it increases her profit and, thus, the utilitarian welfare. If the minimum wage lies below κ_2, an NCC would reduce the profit (as the negative idleness effect outweighs the incentive effect). For minimum wages between κ_2 and κ_3, an NCC increases the profits and the utilitarian welfare. NCCs mitigate the inefficiency that accompanies minimum wages.

For minimum wages above κ_3, the agent gets a rent in the benchmark without NCCs. Moreover, the utilitarian welfare in the benchmark is constant as the equilibrium effort is constant. The comparison of the utilitarian welfare is ambiguous. For minimum wages slightly above κ_3, an NCC improves the utilitarian welfare: It does so for the minimum wage of κ_3 and the incentive and the idleness effect are continuous in the minimum wage. If the minimum wage increases, as argued above, both the incentive and the idleness effect eventually become negative. Therefore, there is a minimum wage above which the utilitarian welfare is smaller with an NCC. The position of this minimum wage depends on the functional form of the effort cost.

5.2 Pareto dominance

As the utilitarian welfare does not consider the distribution of the social surplus, it is maximized without a minimum wage. Thus, the existence of a minimum wage hints at the policymaker's putting weight on the distribution. Therefore, we also compare equilibrium outcomes using Pareto domination. This welfare criterion is relatively uncontroversial as it remains agnostic about how the policymaker aggregates profits and rents in her welfare measure. An equilibrium outcome strictly Pareto dominates another if both the agent's rent and the principal's profit are strictly larger; it weakly Pareto dominates another if either rent or profit are strictly larger and the other one is equal.

An equilibrium outcome that weakly Pareto dominates another also has a strictly larger utilitarian welfare. Hence, for minimum wages between κ_2 and κ_3, the outcome with an NCC weakly Pareto dominates the benchmark. For minimum wages above κ_3, Pareto dominance has no bite as the principal is better but the agent is worse off with an NCC.

Extensive margin There might, however, be a weak Pareto improvement and, thus, efficiency gain on the extensive margin. In all of the above, we have assumed that the principal wants to offer a contract to the agent irrespective of the minimum wage. That is, the success payoff is large enough such that the profit exceeds the principal's outside option for all minimum wages, with or without an NCC. For this paragraph, we drop this simplifying assumption.

Whenever the optimal contract includes an NCC, the principal's profit is strictly larger than in the benchmark. As the possibility to use an NCC weakly increases the principal's profit, the minimum wage at which the principal stops offering a contract to the agent and resorts to her outside option is weakly smaller in the benchmark than with an NCC. It is strictly smaller if the principal's profit at a minimum wage of κ_2 is strictly positive. The principal, hence, offers the agent a contract for more minimum wages, if she can use NCCs. For all minimum wages for which the principal does not participate in the benchmark but does participate with NCCs, the NCC leads to a weak Pareto improvement: The agent gets his outside utility in both cases, whereas the principal makes a profit that exceeds her outside option.

Bounded non-compete clauses When NCCs are bounded, minimum wages can again redistribute from the principal to the agent. If the minimum wage increases, the profit maximizing contract eventually has a constant bonus wage and NCC that lies at the bound (Proposition 5). If the minimum wage increases further, the utilitarian welfare remains constant as in the benchmark for minimum wages above κ_3. In this area, a one unit increase of the minimum wage reduces the principal's profit by one unit and increases the agent's rent by one unit. Because of the NCC, this particular minimum wage is larger than in the benchmark.

It is prohibitively complicated to analytically examine the welfare effects of bounds on NCCs in general. Therefore, we use the fact that both in the benchmark and with bounded NCCs the utilitarian welfare is constant above a specific minimum wage and that minimum wages then redistribute. For an exemplary effort cost function, we show that the constant utilitarian welfare can be larger with bounded NCCs than in the benchmark, if the bound is suitably chosen. This implies that, setting the minimum wage correspondingly, bounded NCCs can lead to outcomes that strictly Pareto dominate any benchmark outcome.

We reconsider the functional form of the cost function and the parameters that we have plotted above: $c(e) = -\ln(1-e) - e$ and $V = 10$. The simplest example relies on the peculiar fact that the principal coincidentally induces first-best effort at κ_4; that is without a bonus wage, using only an NCC of $-V$. We choose this NCC as the bound, $\underline{\bar{v}} = -V$. This implies that the agent starts getting a rent at a minimum wage of κ_4 and that the equilibrium effort remains constant at the first-best level for all larger minimum wages. As the effort is at the first-best level, the incentive effect is maximized and constant in the minimum wage. The inefficiency of the minimum wage without NCCs due to reduced effort is canceled out. This leaves only the inefficiency from the idleness effect, which is also constant in the minimum wage because the equilibrium effort and the NCC are constant in the minimum wage. Thus, with the logarithmic cost function and the bound $\underline{\bar{v}} = -V$ for $\underline{w} \geq \kappa_4$, the utilitarian efficiency is $(1 - e^{FB}) \cdot V$ below the first-best utilitarian efficiency (that is achieved without a minimum wage).

Consider now the utilitarian efficiency in the benchmark for minimum wages above κ_3; minimum wages for which there is redistribution from the principal to the agent. There is one source of inefficiency: too low effort. The equilibrium effort is the same for all minimum wages above κ_3, thus, the utilitarian efficiency is also the same. It is $\int_{e_3^{BM}}^{e^{FB}} V - c'(x)\ dx$ below the first-best.

We can now compare the constant levels of utilitarian welfare with bounded NCCs and in the benchmark. With a bounded NCC, the idleness effect reduces the utilitarian welfare by $(1 - e^{FB}) \cdot V$ compared to the first-best. In the benchmark, the inefficiently low equilibrium effort reduces the utilitarian welfare by $\int_{e_3^{BM}}^{e^{FB}} V - c'(x)\ dx$ compared to the first-best. When V is large enough, the outcome with a bounded NCC is more efficient![16] This is illustrated by Figure 6. On the x-axis, the minimum wage is plotted. On the y-axis, the principal's expected profit is plotted. The utilitarian welfare is equal to the profit and decreasing up to κ_3 for the benchmark and up to κ_4 for the examplary bounded NCC. Beyond the respective thresholds, the utilitarian efficiency is constant; these levels are marked by dotted horizontal lines. The slope of the principal's profit is -1 in both cases; the agent's rent is the remainder between the utilitarian efficiency and the principal's profit. For comparison, we added the case of unbounded NCCs. In this case, the agent never gets a rent and the principal's profit always is the utilitarian efficiency. This illustrates that the utilitarian welfare decreases fast when the equilibrium effort gets close to one because of the Inada conditions.

We now argue that whenever the constant utilitarian welfare is larger with a bounded NCC than in the benchmark, as in the example, a Pareto dominating equilibrium outcome can be constructed. Intuitively, the pie that can be redistributed is the constant utilitarian welfare and the minimum wage determines the agent's rent. In the benchmark, for minimum wages above κ_3, the agent's rent is $\underline{w} - \kappa_3$. With the exemplary bounded NCC, for minimum wages above κ_4, the agent's rent is $\underline{w} - \kappa_4$. Whenever the pie is larger, both principal and agent can be made strictly better off by choosing the minimum wage accordingly. To give the agent the same rent as in the benchmark, minimum wages have to be increased; in this example by $\kappa_4 - \kappa_3$. This procedure can be exported to all other effort cost functions, success payoffs, and bounds by replacing κ_4 by the minimum wage at which the utilitarian welfare becomes constant.

It is difficult to make general statements on the welfare effects of bounded NCCs. There is an interplay between the effort cost function and the bound. Furthermore, equilibrium effort levels that change in the minimum wage further complicate everything. Our example, however, shows that NCCs might be a tool to mitigate the inefficiency that is associated with redistribution in agency problems. Without NCCs, the equilibrium effort is inefficiently low. With bounded NCCs, some social surplus is burned by the NCC, but the equilibrium effort is larger, which potentially makes the social surplus larger. When setting the minimum wage,

[16] When $V \to 0$, both social losses go to zero. The loss with a bounded NCC is coincidentally equal to e^{FB}; it is concave in V. The loss in the benchmark is a more complicated expression, $\sqrt{1+V} - 1 - \frac{1}{2} \cdot \ln(1+V)$. It is the area between V and the marginal cost in the range from e_3^{BM} to e^{FB}; it is convex in V. When increasing V, the loss with a bounded NCC increases initially faster than the loss in the benchmark. For larger V, the loss in the benchmark increases faster. Numerically, they intersect at $V \approx 7.873$.

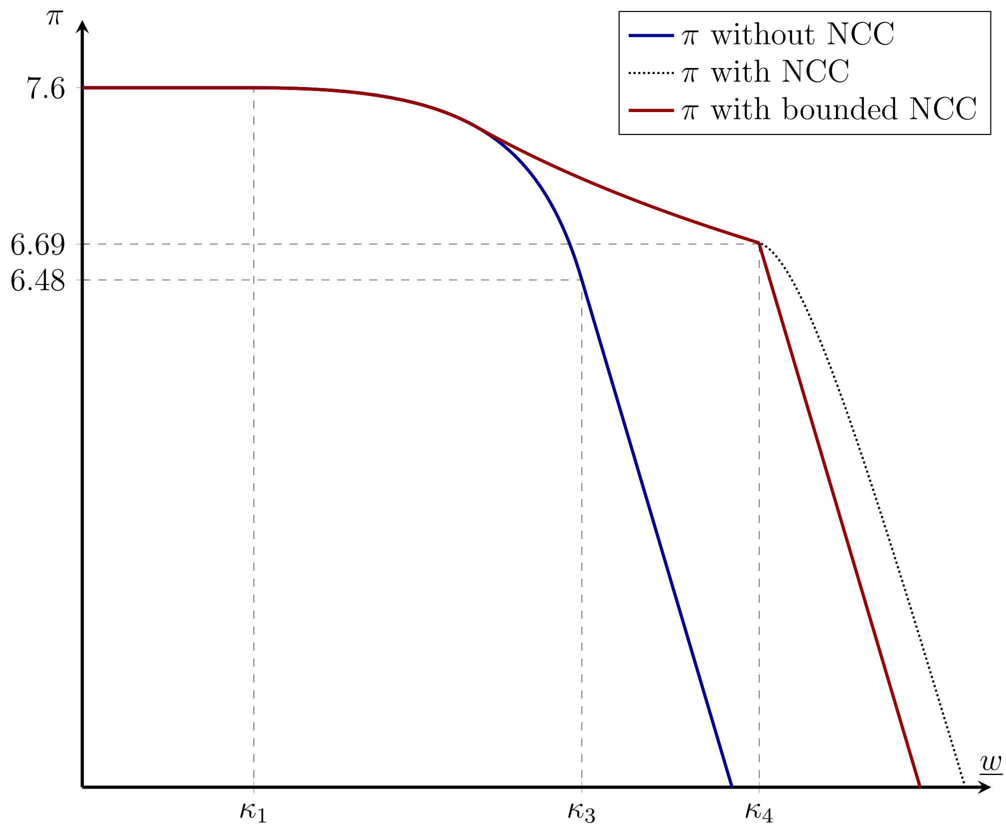

Figure 6: Bounded Non-Compete Clauses Potentially Allow for Strict Pareto Improvements. We choose $c(e) = -\ln(1-e) - e$, $V = 10$ and $\underline{\bar{v}} = -10$.

the interactions of non-compete clauses with the minimum wage have to be taken into account: Larger minimum wages are needed for redistribution when bounded NCCs can be used. There remains the caveat that the minimum wage's redistributory effect strongly depends on the effort cost function that is unknown in the real world. Heterogeneity in agents could make it impossible to find a minimum wage that suits all. Due to all these uncertainties, an ambiguity averse (in a maxmin preferences sense) policymaker might prefer to ban NCCs.

6 Discussion

In this section, we argue that some key aspects of our model are not as restrictive as they seem at first sight. These entail our partial market setting and the continuation payoff. Furthermore, we derive empirical predictions from our model that future work could take to the data.

Partial Market Setting. Because we restrict to a partial market setting, we assume that an increase in the minimum wage does not change the outside option, which we normalize to 0. We argue that this assumption is insubstantial. Firstly, it is not clear that or how the outside option would change in a general equilibrium as there are two effects: On the intensive margin, having a job in a sector in which NCCs are not enforceable and in which the minimum wage is paid gets better when the minimum wage increases. On the extensive margin, however, this other sector might get smaller as its firms react to increased minimum wages by demanding less labor. A smaller outside sector means that the probability of finding employment there reduces, which reduces the outside option. Which of the two effects prevails depends on the modelling assumptions. Secondly, our results continue to hold as long as an increase in the minimum wage does not increase the outside option by the same amount. This is no unrealistic assumption.

One starting point to extend our results to a general equilibrium setting is Acemoglu and Wolitzky (2011) who use a similar principal-agent framework. They use a reduced form matching model to extend their partial market to a general equilibrium and to corroborate their findings from the partial market model.

Continuation Payoff. Besides the partial market setting, we simplify our model by considering only one period and restricting to an ad-hoc continuation payoff. As argued above, this assumption is innocuous if the future entails no moral hazard problems. If the future has moral hazard problems and an ending, the role of NCCs might be weakened, however, it does not change qualitatively.

Consider a slightly more general model with two periods and a moral hazard problem in each. Importantly, the agent retires after the second period. As the agent will not look for a new job, the NCC has no effect in the second period. This model only differs from the presented model if the minimum wage is sufficiently large to grant the agent a rent in the second period, $\underline{w} > \kappa_3$.

We start with looking at the benchmark without NCCs. The agent's earning a rent in the second period allows the principal to use firing threats to provide incentives in the first period even without an NCC. After a bad performance in the first period, the agent can be terminated and denied access to the second period rent (Kräkel and Schöttner 2010 and Schmitz 2005). By using the second-period rent to incentivize the agent, the principal can extract some of the future rent; all of it if the minimum wage is not too high.

An NCC will only be used if the principal cannot already extract all rents without them. If an NCC is used, it converts the remaining rent into incentives in the first period. Because the principal already uses the second period rents to induce more effort in the first period and because of the convex effort cost, however, the incentive effect of the NCC is rather small. Moreover, as the equilibrium effort might already be inefficiently large without an NCC, the NCC could be very wasteful in terms of the social surplus. The qualitative results, however, stay the same.

Empirical predictions. Our model predicts that, everything else equal, an agent with a bad outside option should be more likely to sign an NCC than an agent with a good outside option. Consider a single principal with several agents who work on independent projects. Assume that for various reasons (for example education, age, mobility, or health), these agents have different outside options. In our model, the minimum wage is defined as "minimum wage minus the outside option", because we normalized the outside option. With heterogeneous agents, thus, the same minimum wage is "low" for those with good outside options, and "high" for those with bad outside options. Therefore, we expect to see more and more severe NCCs in the contracts of agents with bad outside options. Surprisingly, those who would have trouble finding a new job anyway are predicted to be bound by NCCs.

Concerning the wages, our model makes ambiguous predictions about the effect of NCCs. For an illustration of the predictions, revisit Figure 3. The principal starts using NCCs at the minimum wage κ_2. At minimum wages slightly above κ_2, the principal has to incre0ase the bonus wage to keep the participation constraint satisfied. Because the NCC also increases the probability of a success, the agent receives this larger bonus more often, implying larger (total) wages than without an NCC. For larger minimum wages, the prediction might reverse. Without NCCs, the bonus wage remains constant for minimum wages larger than κ_3. At some minimum wage, the bonus wage with NCCs falls below this constant level. As soon as the larger probability of a success does not overcompensate the smaller size of the bonus wage anymore, the (total) wage is smaller with an NCC than without. Empirical research could test whether with NCCs there is more incentive pay for low minimum wages and less incentive pay for high minimum wages.

Furthermore, our model predicts that the wages are actually not that informative for the well-being of employees. Although they might receive higher wages, the agent loses his rent due to an NCC because he has to exert more effort. Imagine that there are two states with the same minimum wage that lies above κ_3 such that the agent in the benchmark gets a rent. Those two states differ in their enforcement of NCCs: The first state does not enforce NCCs at all, whereras the second state does. Our model predicts that minimum wage workers in the first state should be happier than those in the second state. The reason is that without an NCC, workers earn a rent, whereas with an NCC the worker's rent gets eaten up by an increased effort.[17]

On the macro level, our model predicts that the effect of minimum wages on the employment is lower when NCCs can be used. Both with bounded or unbounded NCCs, the principal makes weakly larger profits. Therefore, when NCCs can be used, there should be fewer market exits in general. Johnson and Lipsitz (2020) derive the same hypothesis and test it in their Section V. They interact the enforceability measure of Bishara (2011) with the (logarithm of the) minimum wage to check whether access to NCCs moderates the employment effects of a minimum wage. They find a significant and robust effect that supports the hypothesis.

7 Conclusion

Non-compete clauses not only secure returns to investments, conserve bad bargaining positions, or protect proprietary information. They can also provide incentives by the (implicit) threat of being dismissed. In contrast to bonus wages, this use of NCC reduces the agent's payoff and

[17]Measures other than happiness that are interesting might be (self-reported) effort at work or stress-related health issues. In the fast food industry, work effort of minimum-wage workers could be measured as cleanliness, customer satisfaction with the service (or amount of complaints), customer waiting time (or number of sales during peak hours).

makes the participation constraint tight. As a result, the NCC transfers utility from the agent to the principal via an increased equilibrium effort. If NCCs are unbounded, the agent never gets a rent. Because money is a better means to transfer utility, the principal only uses NCCs if a minimum wage constrains her wage setting. This explains both why NCCs are prevalent in low-wage jobs and why an increase in the minimum wage leads to more NCCs.

In the real world, NCCs are bounded. Courts refuse to enforce NCCs that run for an unreasonably long time span. Furthermore, NCCs only prevent employment in the same industry. That is, they can only take the industry-specific human capital hostage.

The welfare analysis shows that an equilibrium with bounded NCCs might strictly Pareto dominate equilibria with only minimum wages and no NCCs. This, however, relies on three points: The effort cost function needs to have a suitable shape such that the loss in the utilitarian welfare with bounded NCCs and minimum wages is lower than the loss from minimum wages alone. Then, the bound on NCCs has to be chosen optimally to realize this improved utilitarian welfare. Lastly, the minimum wage has to be set in the range that makes both the principal and the agent better off: The redistributional effect of the minimum wage only arises at higher levels if bounded NCCs can be used.

In general, policymakers do not have all of this information. Furthermore, the shape of the effort cost function might not allow Pareto improvements. Heterogeneity of the agents complicates things further. As a result, banning NCCs for low-wage workers might be the best option.

A Proofs

A.1 Proof of Proposition 1

Proof of Proposition 1. First, we show that the objective function is strictly concave in the bonus wage. Let $E(b)$ be the maximizer of the agent's utility, that is, the equilibrium effort.

$$E(b) = \begin{cases} (c')^{-1}(b) & \text{if } b \geq 0 \\ 0 & \text{if } b < 0 \end{cases} \tag{8}$$

If the bonus wage is non-negative, the equilibrium effort is determined by the solution of the agent's first-order condition. Furthermore, $E(b)$ is strictly increasing in this range. If the bonus wage is negative, a corner solution, $E(b) = 0$, is optimal. We will use this function with a different argument again, when NCCs are allowed. The first and second derivative of $E(b)$ with respect to its positive argument are $E'(b) = \frac{1}{c''(E(b))}$ and $E''(b) = -\frac{c'''(E(b))}{(c''(E(b)))^3}$.

Remember that the expected profit is $\pi = -w + E(b) \cdot (V - b)$. The first and second derivatives with respect to the bonus wage are then given by:

$$\frac{\partial \pi}{\partial b} = E'(b) \cdot (V - b) - E(b) \tag{9}$$

$$\frac{\partial^2 \pi}{\partial b^2} = E''(b) \cdot (V - b) - 2E'(b) \tag{10}$$

Since $E''(\cdot) < 0$ and $E'(\cdot) > 0$, the second derivative is negative. This implies that P's objective function is strictly concave in the bonus wage.

Next, we look at the constraints of P's problem. We now show that $MWC2$ is always slack. Assume to the contrary that $MWC2$ binds. Rearranging $MWC2$ yields $b = \underline{w} - w$. By $MWC1$ we know that $w \geq \underline{w}$, which then implies that $b \leq 0$. A non-positive bonus wage, however, implies that the equilibrium effort is zero, which cannot be optimal.[18] Hence, $MWC2$ is always slack.

This leaves two constraints that can either bind or be slack, the PC and $MWC1$. We now show that it cannot be the case that both PC and $MWC1$ are slack. Assume to the contrary that both PC and $MWC1$ are slack. This means that there is a profitable deviation: Decreasing w by ϵ still leaves PC and $MWC1$ slack but increases P's expected profit. Therefore, we can decrease w until either PC or $MWC1$ binds.

This leaves us with the following three possible cases:
Case 1: PC binds and $MWC1$ is slack.
Case 2: PC binds and $MWC1$ binds.
Case 3: PC is slack and $MWC1$ binds.

[18] The maximum profit is zero for negative minimum wages and $-\underline{w}$ for positive minimum wages. As we assume that the success payoff is sufficiently large for the principal to be able to achieve a positive profit, a non-positive bonus wage cannot be optimal.

Next, we focus on each case in more detail.

Case 1 P's problem is given by:

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{11}$$

$$\text{subject to} \qquad w + E(b) \cdot b - c(E(b)) = 0 \qquad \text{(PC)}$$

$$w > \underline{w} \quad \text{and} \quad w + b > \underline{w} \qquad \text{(MWC1) and (MWC2)}$$

We will ignore the slack constraints for the moment and later check for which minimum wages they are not violated. The PC can be rewritten such that $E(b) \cdot b = c(E(b)) - w$. We plug this into P's objective function and maximize over the equilibrium effort instead of the bonus wage. The first-order condition is $V = c'(E(b)) = b$. Since the objective function is concave, we know that the first-order condition yields the global maximum. Therefore, $b = V$, $E(V) = e^{FB}$, and $w = c(e^{FB}) - e^{FB} \cdot c(e^{FB})$. Now, we check the constraints. Because $V > 0$, $MWC2$ is slack. $MWC1$ is slack if $\underline{w} < c(e^{FB}) - e^{FB}c'(e^{FB}) \equiv \kappa_1$.

Case 2 P's problem is given by:

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{12}$$

$$\text{subject to} \qquad w + E(b) \cdot b - c(E(b)) = 0 \qquad \text{(PC)}$$

$$w = \underline{w} \quad \text{and} \quad w + b > \underline{w} \qquad \text{(MWC1) and (MWC2)}$$

There are two unknowns and two binding constraints. Plugging $MWC1$ into PC implicitly characterizes $E(b)$ and the bonus wage. There are three subcases: negative minimum wages, $\underline{w} = 0$, and positive minimum wages.

For each negative $\underline{w}$, there are exactly one b and one $E(b)$ such that the participation constraint binds. The reason is the following: Rearrange the binding participation constraint to get

$$E(b) \cdot b - c(E(b)) = -\underline{w} \tag{13}$$

The left-hand side is the part of the agent's utility that is generated by exerting effort. Graphically, it is the area above an increasing function ($c'(e)$), between 0 and $E(b)$, another increasing function. It is zero for a bonus wage of zero, and is strictly increasing in the bonus wage because $c''(e) > 0$. Therefore, there can be at most one bonus wage for each negative minimum wage such that this holds. Furthermore, for negative minimum wages, there is a bijection between b and $E(b)$. Since the right-hand side is strictly positive, so is the bonus wage, which implies $MWC2$.

Consider the minimum wage $\underline{w} = 0$. Since the right-hand side of equation (13) is zero, so is the equilibrium effort, which means that the bonus wage has to be non-positive. $MWC2$ is only slack if the bonus wage is positive. Thus, there is no bonus wage such that PC binds and $MWC2$ is slack.

Consider positive minimum wages. The participation constraint is always slack. That is, there are no bonus wage and no equilibrium effort that satisfy equation (13).

Summing up the optimal contract in Case 2: For negative minimum wages, let $e_2^{BM}(\underline{w})$ denote the effort that makes the participation constraint (13) binding. Then, $e_2^{BM}(\underline{w})$ is implicitly defined by $e_2^{BM}(\underline{w}) \cdot c'(e_2^{BM}(\underline{w})) - c(e_2^{BM}(\underline{w})) = -\underline{w}$. We also get that $b = c'(e_2^{BM}(\underline{w}))$ and from $MWC1$ we get $w = \underline{w}$.

Case 3 P's problem is given by:

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{14}$$

$$\text{subject to} \qquad w + E(b) \cdot b - c(E(b)) > 0 \tag{PC}$$

$$w = \underline{w} \quad \text{and} \quad w + b > \underline{w} \qquad \text{(MWC1) and (MWC2)}$$

We will ignore the slack constraints for the moment and later check for which minimum wages they are not violated. We plug $MWC1$ into the objective function and take the derivative. The optimal bonus wage is characterized by the marginal profit's being 0. The solution to the first-order condition implicitly defines the optimal effort in Case iii, e_3^{BM}: $c'(e_3^{BM}) + e_3^{BM} \cdot c''(e_3^{BM}) = V$. Hence, $e_3^{BM} < e^{FB}$. We also get that $w = \underline{w}$ and $b = c'(e_3^{BM})$. Next, we check the constraints. As $e_3^{BM} > 0$, $MWC2$ is slack. PC is slack if $\underline{w} > c(e_3^{BM}) - e_3^{BM} c'(e_3^{BM}) \equiv \kappa_3$.

The Optimal Contract We have verified that the optimal contract from Case 1 is feasible if $\underline{w} < \kappa_1$, the optimal contract from Case 2 is feasible if $\underline{w} < 0$, and the optimal contract from Case 3 is feasible if $\underline{w} > \kappa_3$. These thresholds are $\kappa_1 = c(e^{FB}) - e^{FB} c'(e^{FB}) < 0$ and $\kappa_3 = c(e_3^{BM}) - e_3^{BM} c'(e_3^{BM}) < 0$. Because $e_3^{BM} < e^{FB}$, it follows that $\kappa_1 < \kappa_3$.

Thus, for $\underline{w} < \kappa_1$, we have two candidates: Case 1 and Case 2. The maximization problem in Case 2 has two binding constraints, while the maximization problem in Case 1 has none. As a result, the profit from the optimal contract in Case 1 is weakly larger. The concavity of the objective function and the fact that the bonus wages from Case 1 and Case 2 are different for all $\underline{w} < \kappa_1$ imply that the profit is strictly larger. For $\kappa_1 \leq \underline{w} \leq \kappa_3$, the only candidate is Case 2; thus, this contract is optimal. For $\kappa_3 < \underline{w}$, we have again two candidates: Case 2 and Case 3. Since the maximization problem in Case 3 has only one binding constraint, the profit from the optimal contract in Case 3 is weakly larger. Again, concavity and different solutions imply strictly larger profits.

□

A.2 Proof of Proposition 2

Proof of Proposition 2. The proof proceeds in two main parts. The first part is about simplifying the problem. Since there are four inequality constraints, there are 16 possible combinations.

First, we identify those four combinations that can be optimal. In all of those combinations, the participation constraint is binding; the agent does not get a rent. We use this fact to reduce the problem's dimensionality by using the participation constraint to express the optimal NCC in terms of the minimum wage and the bonus wage. The first combination is the same as Case 1 in the benchmark, which means that this contract is profit maximizing for $\underline{w} < \kappa_1$. For all $\underline{w} \geq \kappa_1$, the base wage has to be the minimum wage. This fact and an additional piece of notation simplify the problem further. This yields a strictly quasi-concave objective function in only the bonus wage with one inequality constraint. The optimal bonus wage and whether the inequality constraint binds shows into which combination the contract falls. In the second part, we solve this rewritten problem.

The Possibly Optimal Combinations The agent's first-order condition for the optimal effort with NCCs is

$$b - \bar{v} = c'(e). \tag{15}$$

Whenever the left-hand side is non-negative, the first-order condition yields the optimal equilibrium effort, which we, express as $E(b - \bar{v}) \equiv (c')^{-1}(b - \bar{v})$. As above, a negative left-hand side implies that the corner solution $E(b - \bar{v}) = 0$ is optimal.

The principal's problem is

$$\begin{aligned} \max_{w,b,\bar{v}} \quad -w+ \quad & E(b-\bar{v}) \cdot (V - b) && (16) \\ \text{subject to} \quad & w + E(b-\bar{v}) \cdot b + (1 - E(b-\bar{v})) \cdot \bar{v} - c(E(b-\bar{v})) \geq 0 && \text{(PC)} \\ & \bar{v} \leq 0 && \text{(NCC)} \\ & w \geq \underline{w} \qquad w + b \geq \underline{w} && \text{(MWC1) and (MWC2)} \end{aligned}$$

To solve the principal's problem, one has to know which constraints bind and which are slack for different minimum wages. In total, there are 16 combinations. They are summarized in Table 1. The combinations' order in Figure 2 reflects their occurrence when the minimum wage increases. We will now prove that the optimal contract always falls into the Combinations 1 to 4 and never into the Combinations 5 to 16 (column six of Table 1) for three distinct reasons.

Firstly, the participation constraint has to bind. Otherwise, there is a profitable deviation: Make the NCC more severe, keeping everything else fixed. Note that the bonus wage is optimally never larger than the success. Then, the agent exerts more effort which leads to more successes and more profit.

Secondly, it cannot be that $MWC2$ and NCC bind simultaneously. If they did, the agent would exert no effort. Then, the principal has no revenue. This cannot be optimal by our assumption that the success payoff is sufficiently large to allow for positive profits.

Thirdly, $MWC1$ can only be slack when the NCC feasibility constraint binds. Otherwise, there is a profitable deviation. In these combinations, the principal uses an NCC and pays a

No.	PC	NCC	MWC1	MWC2	Relevant?
1	binds	binds	slack	slack	$\underline{w} \leq \kappa_1$
2	binds	binds	binds	slack	$\kappa_1 < \underline{w} \leq \kappa_2$
3	binds	slack	binds	slack	$\kappa_2 < \underline{w} \leq \kappa_4$
4	binds	slack	binds	binds	$\kappa_4 < \underline{w}$
5	slack	binds	binds	binds	no, PC
6	slack	binds	binds	slack	no, PC
7	slack	binds	slack	binds	no, PC
8	slack	binds	slack	slack	no, PC
9	slack	slack	binds	binds	no, PC
10	slack	slack	binds	slack	no, PC
11	slack	slack	slack	binds	no, PC
12	slack	slack	slack	slack	no, PC
13	binds	binds	binds	binds	no, no effort
14	binds	binds	slack	binds	no, no effort
15	binds	slack	slack	binds	no, deviation
16	binds	slack	slack	slack	no, deviation

Table 1: The 16 Combinations of Binding Constraints.

larger than necessary base wage. This cannot be optimal because there is a profitable deviation: Decrease the base wage by one unit and increase the bonus wage and make the NCC less severe by one unit. Because bonus wage and the NCC's severity are perfect substitutes, the equilibrium effort stays the same. Furthermore, the participation constraint remains satisfied: The agent loses one unit on the base wage but gains one unit both if there is a success and if there is a failure. The principal's profit increases because he saves on the base wage one unit with certainty and loses on the bonus wage one unit with the success probability (less than one by the Inada conditions). The principal can repeat this deviation until either MWC1 or NCC binds.

When is the first combination optimal? In the benchmark, we have seen that in the first combination the optimal contract implements the first-best effort. Additionally, the principal extracts the whole surplus. Therefore, this contract is profit-maximizing whenever it is feasible.

As we have seen in the benchmark, the contract in the first combination is only feasible if $\underline{w} < \kappa_1 = c(e^{FB}) - e^{FB}c'(e^{FB}) < 0$. This implies that for all $\underline{w} \geq \kappa_1$, the optimal contract is from either the second, the third, or the fourth combination. In all of these combinations, the base wage optimally is the minimum wage; $MWC1$ binds.

From now on, $\underline{w} \geq \kappa_1$, which eliminates w from the problem. Thus, b and $\bar{v}$ remain. Furthermore, the participation constraint PC has to bind. This lets us express $\bar{v}$ as an implicit function of $\underline{w}$ and b:

$$\bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))}. \tag{17}$$

Note that $(b - \bar{v})$ is non-negative because of $MWC1$ binds, which simplifies $MWC2$ to $b \geq 0$ and NCC, $\bar{v} \leq 0$. Thus, the agent's first-order condition yields the equilibrium effort.

$\bar{v}(\underline{w}, b)$ is the most severe NCC that the agent is willing to accept given a base wage of $\underline{w}$ and a bonus wage b. Lemma 6 shows that the higher the minimum wage is, the more severe is this NCC for a given bonus wage. The higher the bonus wage is, the more severe is this NCC for a given minimum wage. Furthermore, due to monotonicity, the values of $\bar{v}(\underline{w}, b)$ are unique in b for a fixed $\underline{w}$ and the other way around.

Therefore, the principal's problem can also be expressed as

$$\begin{aligned} \max_{b} \quad & -\underline{w} + E(b - \bar{v}(\underline{w}, b))\,(V - b) && (18)\\ \text{subject to} \quad & \bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))} && (\text{PC'})\\ & \bar{v} \leq 0 && (\text{NCC})\\ & b \geq 0 && (\text{MWC2}) \end{aligned}$$

Whenever $\underline{w} \geq \kappa_1$, a contract is optimal if and only if it solves the simplified problem. In the second combination, $MWC2$ is slack and NCC binds. In the third combination, $MWC2$ and NCC are both slack. In the fourth combination, $MWC2$ binds and NCC is slack.

Lemma 6. *i) Fix a minimum wage. The NCC that makes the participation constraint bind $\bar{v}(\underline{w}, b)$ is strictly decreasing in the bonus wage: $\frac{\partial \bar{v}(\underline{w}, b)}{\partial b} < 0$.*
ii) Fix a bonus wage. The NCC that makes the participation constraint bind $\bar{v}(\underline{w}, b)$ is strictly decreasing in the minimum wage: $\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} < 0$.

Proof. Rearrange the binding participation constraint to

$$Z \equiv \underline{w} + E(b - \bar{v}) \cdot (b - \bar{v}) + \bar{v} - c(E(b - \bar{v})) = 0, \tag{19}$$

Because this is continuously differentiable, the implicit function theorem can be used to get the derivatives of $\bar{v}$ with respect to $\underline{w}$ and b.

$$\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} = -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial \bar{v}}} = -\frac{1}{-E'(b - \bar{v}) \cdot (b - \bar{v}) - E(b - \bar{v}) + 1 + c'(E(b - \bar{v})) \cdot E'(b - \bar{v})} \tag{20}$$

$$= -\frac{1}{1 - E(b - \bar{v})} \tag{21}$$

The simplification is due to the agent's first-order constraint, $(b - \bar{v} - c'(E)) = 0$.

$$\frac{\partial \bar{v}(\underline{w}, b)}{\partial b} = -\frac{\frac{\partial Z}{\partial b}}{\frac{\partial Z}{\partial \bar{v}}} = -\frac{E'(b-\bar{v}) \cdot (b-\bar{v}) + E(b-\bar{v}) - c'(E(b-\bar{v})) \cdot E'(b-\bar{v})}{-E'(b-\bar{v}) \cdot (b-\bar{v}) + 1 - E(b-\bar{v}) + c'(E(b-\bar{v})) \cdot E'(b-\bar{v})} \tag{22}$$

$$= -\frac{E(b-\bar{v})}{1-E(b-\bar{v})} \tag{23}$$

Again, the agent's first-order constraint simplifies the expression. □

We will now define a useful term to simplify the maximization problem further. Let $b_2^{**}(\underline{w})$ denote the optimal bonus wage in Case 2 of the benchmark (binding PC, binding $MWC1$, slack $MWC2$). The case conditions imply a property of $b_2^{**}(\underline{w})$: It makes the participation constraint binding in the absence of an NCC.

To use this particular bonus wage to simplify the problem, we have to extend the definition of $b_2^{**}(\underline{w})$ to minimum wages above κ_3 for which it is not the optimal bonus wage. Let $b_2^{**}(\underline{w})$ denote the *minimum non-negative* bonus wage that keeps the participation constraint *satisfied* in the absence of an NCC.

$$\forall \underline{w} \geq \kappa_1 \qquad b_2^{**}(\underline{w}) \equiv \min\left\{b \in \mathbb{R}_0^+ \;\middle|\; \underline{w} + E(b) \cdot b - c(E(b)) \geq 0\right\} \tag{24}$$

For non-positive minimum wages, $b_2^{**}(\underline{w})$ is determined by the minimum wage that makes the participation constraint binding. For positive minimum wages the participation constraint is always slack without an NCC; there is no bonus wage that makes the participation constraint binding. Thus, if $\underline{w} \geq 0$, then $b_2^{**}(\underline{w}) = 0$. Furthermore, $b_2^{**}(\underline{w})$ has the nice property that it exists and it is strictly decreasing in the minimum wage between κ_1 and 0.

To simplify the problem, we now replace the inequality constraints using $b_2^{**}(\underline{w})$: As long as PC' holds, the conditions NCC and $MWC2$ are equivalent to another condition, $b \geq b_2^{**}(\underline{w})$.

Consider $\underline{w} < 0$. In this case, PC' and NCC imply $MWC2$. The bonus wage has to be at least $b_2^{**}(\underline{w})$, even without an NCC, to satisfy the participation constraint. If $\underline{w} < 0$, then $b_2^{**}(\underline{w}) > 0$, implying $MWC2$. In this case, the new constraint $b \geq b_2^{**}(\underline{w})$ is binding if and only if NCC is binding.

Consider $\underline{w} \geq 0$. In this case, PC' and $MWC2$ imply NCC. If $\underline{w} \geq 0$, then $b_2^{**}(\underline{w}) = 0$; for $\underline{w} = 0$ the participation constraint is binding without an NCC, for $\underline{w} > 0$, the participation constraint is slack without an NCC. In both cases, the binding PC means that $\bar{v} \leq 0$, implying NCC. In this case, the new constraint is binding if and only if $MWC2$ is binding.

The problem is, thus, equivalent to

$$\max_{b} \quad -\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot (V - b) \tag{25}$$

$$\text{subject to} \qquad \bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))} \tag{PC'}$$

$$b \geq b_2^{**}(\underline{w}). \tag{26}$$

The problem (25) is simpler because it has only one inequality constraint which is on the only argument of the objective function. Under the assumptions made in Section 3, moreover, the objective function is strictly concave, as Lemma 7 shows. We introduced this assumption because it implies all assumptions that we need in this proof. To make the proof tighter, however, we make weaker assumptions wherever possible. Thus, for determining whether the second or the third combination is optimal, we will use a weaker assumption and the notion of strict quasi-concavity that is sufficient to derive the results. In Lemma 8, we determine the necessary and sufficient condition that makes the objective function strictly quasi-concave in the bonus wage.

Lemma 7. *(25) is strictly concave in b if for all bonus wages*

$$\frac{c'''(E(b, \bar{v}(\underline{w}, b)))}{c''(E(b, \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b, \bar{v}(\underline{w}, b))}. \tag{27}$$

Proof. The objective function's first and second derivatives with respect to the bonus wage are

$$\frac{\partial \pi}{\partial b} = \frac{E'(b, \bar{v}(\underline{w}, b))}{1 - E(b, \bar{v}(\underline{w}, b))} \cdot (V - b) - E(b, \bar{v}(\underline{w}, b)) \tag{28}$$

and (omitting the argument of $E(b, \bar{v}(\underline{w}, b))$ for readability)

$$\frac{\partial^2 \pi}{\partial b^2} = \left[\frac{E''}{(1-E)^2} + \frac{(E')^2}{(1-E)^3}\right] \cdot (V - b) - \frac{2E'}{1-E}. \tag{29}$$

Because $E'(b, \bar{v}(\underline{w}, b)) > 0$, a sufficient condition for the concavity of the objective function is that $\frac{E''}{(1-E)^2} + \frac{E'E'}{(1-E)^3} < 0$. Rearranging and simplifying shows that this is true under our assumption on the cost function.

$$E''(b, \bar{v}(\underline{w}, b)) + \frac{(E'(b, \bar{v}(\underline{w}, b)))^2}{1 - E(b, \bar{v}(\underline{w}, b))} < 0 \quad \implies \quad \frac{\partial^2 \pi}{\partial b^2} < 0 \tag{30}$$

Plugging in for $E'(\cdot) \equiv \frac{1}{c''(E(\cdot))}$ and $E''(\cdot) \equiv -\frac{c'''(E(\cdot))}{(c''(E(\cdot)))^3}$ yields

$$\frac{c'''(E(b, \bar{v}(\underline{w}, b)))}{c''(E(b, \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b, \bar{v}(\underline{w}, b))} \tag{31}$$

□

Lemma 8. *(25) is strictly quasi-concave in b if for all bonus wages*

$$\frac{c'''(E(b - \bar{v}(\underline{w}, b)))}{c''(E(b - \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b - \bar{v}(\underline{w}, b))} - \frac{2}{E(b - \bar{v}(\underline{w}, b))}. \tag{32}$$

Proof. The objective function, $\pi(\underline{w}, b)$, is twice continuously differentiable. It is strictly quasi-concave in b if the second derivative is negative at each critical point.

For readability, we will omit the argument of $E(b - \bar{v}(\underline{w}, b))$ and its derivatives, and instead write $E(\cdot)$. The objective function's first derivative with respect to b is

$$\begin{aligned}\frac{\partial \pi(\underline{w}, b)}{\partial b} &= E'(\cdot) \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b)}{\partial b}\right) \cdot (V - b) - E(\cdot) \\ &= \frac{E'(\cdot)}{1 - E(\cdot)} \cdot (V - b) - E(\cdot). \end{aligned} \tag{33}$$

Since $1-E(\cdot)$ is the equilibrium probability of a failure, it is positive due to the Inada conditions. Critical points are characterized by

$$V - b = \frac{E(\cdot) \cdot (1 - E(\cdot))}{E'(\cdot)}. \tag{34}$$

The objective function is strictly quasiconcave in b if and only if the derivative of equation (33) is negative at every critical point. After some calculus, the sign of the derivative of equation (33) is seen equal to the sign of "expression 1":

$$E'(\cdot) \cdot (V - b) - E(\cdot) \cdot (1 - E(\cdot)). \tag{Expression 1}$$

Expression 1's derivative is

$$\begin{aligned} &E''(\cdot) \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b)}{\partial b}\right) \cdot (V - b) - E'(\cdot) - E'(\cdot) \cdot (1 - 2E(\cdot)) \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b)}{\partial b}\right) \\ &= \frac{E''(\cdot)}{1 - E(\cdot)} \cdot (V - b) - E'(\cdot) - \frac{E'(\cdot) \cdot (1 - 2E(\cdot))}{1 - E(\cdot)} \\ &= \frac{E''(\cdot)}{1 - E(\cdot)} \cdot (V - b) - \frac{E'(\cdot) \cdot (2 - 3E(\cdot))}{1 - E(\cdot)} \end{aligned} \tag{35}$$

Since we only care about the sign at the critical points, we can now plug in the solution of the first-order condition (34) for $(V - b)$. This yields an expression that we would like to be negative.

$$\frac{E''(\cdot)}{1 - E(\cdot)} \cdot \frac{E(\cdot) \cdot (1 - E(\cdot))}{E'(\cdot)} - \frac{E'(\cdot) \cdot (2 - 3E(\cdot))}{1 - E(\cdot)} < 0 \tag{36}$$

Rearranging yields

$$E''(\cdot) < \frac{(E'(\cdot))^2 \cdot (2 - 3E(\cdot))}{E(\cdot) \cdot (1 - E(\cdot))} \tag{37}$$

Using the definition of $E(\cdot)$, this can be simplified.

$$E(\cdot) = (c')^{-1}(\cdot) \quad \Longrightarrow \quad E'(\cdot) = \frac{1}{c''(E(\cdot))} \quad \Longrightarrow \quad E''(\cdot) = -\frac{c'''(E(\cdot))}{(c''(E(\cdot)))^3} \tag{38}$$

Therefore, (37) is equivalent to our assumption

$$\frac{c'''(E(\cdot))}{c''(E(\cdot))} > \frac{1}{1-E(\cdot)} - \frac{2}{E(\cdot)} \tag{39}$$

$\square$

For equilibrium efforts below $\frac{2}{3}$, the assumption is always satisfied. For equilibrium efforts above $\frac{2}{3}$, the assumption says that the marginal cost has to be convex enough. As a result, the equilibrium effort reacts not too strongly to increased incentives and the strict quasi-concavity is preserved when introducing NCCs.

Strict quasi-concavity in the bonus wage implies that the maximum is unique if it exists. To see that the maximum exists, note that the maximum is equivalent to the maximum of the problem constraining $b_2^{**}(\underline{w}) \leq b \leq V$, since the optimal bonus wage cannot be above V. Because of the extreme value theorem we know that the latter problem has a solution ($b_2^{**}(\underline{w}) \leq b \leq V$ is a compact set and the objective function is continuous).

This last simplification concludes the first part of the proof. In the second part of the proof, we look at the three remaining combinations and determine for which minimum wages they are optimal. We first characterize the different combinations in the simplified problem. Then, we use the monotonicity of the marginal profit in the bonus wage evaluated at the bonus wage $b_2^{**}(\underline{w})$ to find the minimum wages for which the second combination is optimal. Lastly, we derive a condition under which the fourth combination is optimal for some minimum wages.

Negative minimum wages Consider negative minimum wages first. For $\kappa_1 \leq \underline{w} < 0$, only the second or the third combination can be optimal. The sign of the derivative of the objective function with respect to the bonus wage at the lower bound $b_2^{**}(\underline{w})$ shows whether there is an inner solution or not. If the derivative is non-positive, there is a corner solution and, thus, no NCC. The second combination is optimal. If the derivative is positive, there is an inner solution and, thus, an NCC. The third combination is optimal. The monotonicity of the derivative evaluated at $b_2^{**}(\underline{w})$ in the minimum wage yields uniqueness of minimum wage at which a switch happens.

Lemma 9. *Assume that* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{2}{E(b-\bar{v}(\underline{w},b))}$ *for all bonus wages. There is a unique cutoff* $\kappa_2 < 0$ *in the minimum wage such that for all* $\kappa_1 \leq \underline{w} \leq \kappa_2$*, the optimal contract has* $b = b_2^{**}(\underline{w})$*, and for all* $\kappa_2 < \underline{w} < 0$*, the optimal contract has* $b > b_2^{**}(\underline{w})$*.*

Proof. The derivative of the profit with respect to the bonus wage evaluated at the lower bound is

$$\begin{aligned}\frac{\partial \pi(\underline{w}, b)}{\partial b}\Big|_{b=b_2^{**}(\underline{w})} &= \frac{\partial E(b-\bar{v}(\underline{w}, b))}{\partial b}\Big|_{b=b_2^{**}(\underline{w})} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \\ &= \frac{E'(b_2^{**}(\underline{w}))}{1-E(b_2^{**}(\underline{w}))} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}).\end{aligned} \tag{40}$$

We will now look at different minimum wages and show that there is exactly one minimum wage at which the optimum switches from a corner to an inner solution. The corresponding minimum wage is the minimum wage from which on NCCs are used, κ_2. Technically, at κ_2, the objective function's first derivative evaluated at the lowest possible bonus wage $b_2^{**}(\underline{w})$ switches the sign from negative (corner solution) to positive (inner solution).

We use the same strategy as when proving quasi-concavity: We show that in all candidates for κ_2, the derivative goes from negative to positive. By continuity, there can be only one candidate.

A candidate for κ_2 is a minimum wage such that the derivative is zero:

$$\frac{\partial \pi(\underline{w}, b)}{\partial b}\Big|_{b=b_2^{**}(\underline{w})} = \frac{E'(b_2^{**}(\underline{w})}{1 - E(b_2^{**}(\underline{w}))} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \stackrel{!}{=} 0 \tag{41}$$

$$\iff \quad (V - b_2^{**}(\underline{w})) = \frac{E(b_2^{**}(\underline{w})) \cdot (1 - E(b_2^{**}(\underline{w})))}{E'(b_2^{**}(\underline{w}))} \tag{42}$$

To see how the derivative of the profit with respect to the bonus wage at the lower bound changes, take the derivative with respect to the minimum wage. Note that although $\bar{v}(b, \underline{w})$ is a function of both the bonus and the minimum wage, it will not change: At $b_2^{**}(\underline{w})$, the participation constraint binds without an NCC. Thus, $\bar{v}(b_2^{**}(\underline{w}), \underline{w}) = 0$ for all negative minimum wages.

Again, we work with another expression that has the same sign as the first derivative but which is easier to work with. "Expression 2" is

$$E'(b_2^{**}(\underline{w})) \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \cdot (1 - E(b_2^{**}(\underline{w}))) \tag{Expression 2}$$

The derivative of expression 2 with respect to the minimum wage (where we express $E(b_2^{**}(\underline{w}))$ and its derivatives as E to improve readability) is

$$\begin{aligned}
\frac{\partial \left(\frac{\partial \pi}{\partial b}\Big|_{b=b_2^{**}(\underline{w})} \right)}{\partial \underline{w}} &= E'' \cdot (V - b_2^{**}(\underline{w})) \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} - E' \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} \\
&\quad - (1 - E) \cdot E' \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} + E' \cdot E \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} \\
&= \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} \left(E'' \cdot \frac{E(1-E)}{E'} - 2E' \cdot (1 - E) \right) > 0
\end{aligned} \tag{43}$$

The second line follows from plugging (42) in. At the critical point, the derivative of the profit with respect to the bonus wage evaluated at the lower bound is increasing because $\frac{\partial b_2^{**}}{\partial \underline{w}} < 0$; the lowest bonus wage to satisfy the participation constraint is decreasing in the minimum wage because a higher minimum wage makes the participation constraint already slack. Moreover, it is globally true that $E' > 0$, and $E'' < 0$.

We have shown that any switches between corner and inner solutions have to be from corner to inner solutions. Moreover, there can be at most one switching point. That is, conditional on existence, κ_2 is unique.

To show that there is at least one critical point, we use that the derivative of the profit with respect to the bonus wage is continuous in the minimum wage. There is a minimum wage for which the derivative is negative and there is a minimum wage for which the derivative is positive. Thus, there also is a minimum wage for which the derivative is zero.

The derivative is negative for the minimum wage κ_1. The principal implements first-best effort and extracts all surplus by selling the firm. Because all of the success payoff goes to the agent, increasing the bonus wage further reduces the profit. Plugging κ_1 in, yields $b_2^{**}(\kappa_1) = V$. The derivative is

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=b_2^{**}(\kappa_1)} = -E(V) < 0. \tag{44}$$

The derivative is positive for the minimum wage κ_3. Following a similar argument as above, we know from the benchmark that the derivative of the profit with respect to the bonus wage without access to NCC at the minimum wage κ_3 is zero: Left of κ_3, the optimal bonus wage just satisfies the participation constraint, right of κ_3, the optimal bonus wage makes the participation constraint slack. The derivative of the profit with respect to the bonus wage without NCC is

$$\left.\frac{\partial \pi^{\text{No NCC}}}{\partial b}\right|_{b=b_2^{**}(\kappa_3),\bar{v}=0} = E'(b_2^{**}(\kappa_3)) \cdot (V - b_2^{**}(\kappa_3)) - E(b_2^{**}(\kappa_3)) = 0 \tag{45}$$

With NCCs, there are double incentives. Thus, the derivative with NCCs is strictly larger: The marginal benefit gets multiplied with $\frac{1}{1-E} > 1$. Therefore, the positive term is larger. The negative term is the same. Since at κ_3 the derivative without NCC is zero, the derivative with NCC is positive.

$$\left.\frac{\partial \pi(\underline{w}, b)}{\partial b}\right|_{b=b_2^{**}(\kappa_3),\bar{v}=0} = \frac{E'(b_2^{**}(\kappa_3))}{1 - E(b_2^{**}(\kappa_3))} \cdot (V - b_2^{**}(\kappa_3)) - E(b_2^{**}(\kappa_3)) > 0 \tag{46}$$

To sum up: The profit's first derivative evaluated at the bonus wage $b_2^{**}(\underline{w})$ is continuous and monotonically increasing. It is strictly negative at κ_1 and strictly positive at κ_3. Thus, its root, κ_2, exists and lies strictly in-between, $\kappa_1 < \kappa_2 < \kappa_3 < 0$.

□

For all minimum wages below κ_2, the optimal contract and, thus, the profit is the same as in the benchmark. For minimum wages above κ_2, an NCC is used and the principal's profits are strictly larger than in the benchmark: Strict quasi-concavity of the profit in the bonus wage means that the maximum is unique. The principal could mimic the world without NCC. He does, however, not want to. Uniqueness of the maximimum means that the optimal contract with NCC is strictly better than the optimal contract without NCC.

A minimum wage of zero For $\underline{w} = 0$, the second combination is not feasible. The binding participation constraint with no NCC implies that the bonus wage has to be zero. In the second combination, the bonus wage has to be strictly positive. Furthermore, the fourth combination is not feasible. The binding participation constraint with no bonus wage implies that the most severe NCC is no NCC. In the fourth combination, the NCC has to be strictly negative. Thus, the optimal contract has to have both a bonus wage and an NCC.

Having established that the first, the second, and then the third combination are optimal in an increasing minimum wage, we now turn to positive minimum wages.

Positive minimum wages For positive minimum wages, contracts from the second combination are not feasible: It is not possible to make the participation constraint binding without an NCC. In this range, only the third or the fourth combination can be optimal. We show that starting at a minimum wage of 0, the third combination is optimal. We derive one condition on the effort cost function for the existence and one condition for the uniqueness of there being a minimum wage $\kappa_4 > 0$ such that for all $\underline{w} < \kappa_4$, the third combination is optimal and for all $\underline{w} \geq \kappa_4$, the fourth combination is optimal. At κ_4, the principal stops using a bonus wage. Instead, all incentives follow from an NCC. If the condition is not met, the third combination is optimal for all positive minimum wages.

To get uniqueness of κ_4, we need an assumption on the cost function. For all bonus wages, it has to hold that $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$. While this assumption is stronger than the assumption to get strict quasi-concavity, it is also implied by our assumptions in Section 3 that imply strict concavity of the objective function. With this assumption, we can show that there is at most one minimum wage at which the principal switches between the third and the fourth combination. Furthermore, this assumption implies that the switch is such that for lower minimum wages there is a positive bonus wage, while for higher minimum wages, the optimal bonus wage is zero.

The strategy of the proof is to determine the sign of the marginal profit of the bonus wage, evaluated at a bonus wage of 0. If it is positive, there is an inner solution and the optimal bonus wage is positive. To make the participation constraint binding, an NCC is needed. The optimal contract is, thus, from the third combination. Using no bonus wage is optimal if it is negative. Then, the first unit of the bonus wage is not worth the marginal cost. The optimal contract is, thus, from the fourth combination. The assumption on the uniqueness implies that the every switch of the sign goes from the positive to the negative.

To prove existence, we show that the sign of the marginal profit of the bonus wage, evaluated at a bonus wage of 0, is initially positive. We assume that the condition for uniqueness is met. The marginal profit of the first unit of bonus wage is continuous in the minimum wage. Because its sign is initially positive, can switch its sign at most once, and the marginal profit's continuity, the sign in the limit is negative if and only if the switch happened for a finite minimum wage. We then derive the (necessary and sufficient) condition under which the sign is negative in the limit. This is the conition for the existence of κ_4. To determine the sign in the limit, we use L'Hôpital's rule.

Lemma 10. *If for all bonus wages* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$*, then there is at most one minimum wage for which* $\frac{\partial\pi}{\partial b}\Big|_{b=0} = 0$

Proof. Again, we will employ the same strategy of proof as above to show the uniqueness of a critical point. The critical point in the minimum wage is characterized by

$$\frac{\partial\pi}{\partial b}\Big|_{b=0} = \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V - E(-\bar{v}(\underline{w},0)) \stackrel{!}{=} 0. \tag{47}$$

This is the marginal profit by increasing the bonus wage starting at a bonus wage of zero. That is, for which minimum wage it is optimal not to use the bonus wage $b = 0$. Since $\underline{w} > 0$, the principal will use an NCC to provide incentives. The optimal contract falls into the fourth combination.

Thus, a critical point is defined by

$$V = \frac{E(-\bar{v}(\underline{w},0)) \cdot (1 - E(-\bar{v}(\underline{w},0)))}{E'(-\bar{v}(\underline{w},0))} \tag{48}$$

As above, we show that this critical point is unique if it implies that the marginal profit from the first unit of bonus wage hits zero from above. Then, to the left of the critical point, it is optimal to use positive bonus wages; to the right of the critical point, it is optimal to use no bonus wages. We want to show that

$$\frac{\partial\pi}{\partial b}\Big|_{b=0} \stackrel{!}{=} 0 \quad \Longrightarrow \quad \frac{\partial\left(\frac{\partial\pi}{\partial b}\Big|_{b=0}\right)}{\partial\underline{w}} < 0 \tag{49}$$

To do so, we compute this derivative (we again omit the arguments and express $E(-\bar{v}(\underline{w},0))$ as E to improve readability)

$$\frac{\partial\left(\frac{\partial\pi}{\partial b}\Big|_{b=0}\right)}{\partial\underline{w}} = \frac{(1-E)E'' + E' \cdot E'}{(1-E)^3} \cdot V - \frac{E'}{1-E} \tag{50}$$

Plugging in the characterization of a critical point ($V = \frac{E\cdot(1-E)}{E'}$) and simplifying yields

$$\frac{c'''(E)}{c''(E)} > \frac{1}{1-E} - \frac{1}{E}, \tag{51}$$

which holds by assumption. □

Lemma 11. *Assume that for all bonus wages* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$*. If*

$$\lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V < 1, \tag{52}$$

then there is a minimum wage $\kappa_4 > 0$ *such that the optimal contract uses a bonus wage for all lower minimum wages and the optimal contract uses no bonus wage for all larger minimum* *wages.*

Proof. κ_4 exists if there is a positive minimum wage that equates the marginal benefit and the marginal cost of the first unit of bonus wage.

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=0} = \frac{E'(-\bar{v}(\underline{w}, 0))}{1 - E(-\bar{v}(\underline{w}, 0))} \cdot V - E(-\bar{v}(\underline{w}, 0)) = 0. \tag{53}$$

We have shown above that there is at most one such minimum wage. Furthermore, we have shown that the intersection has to be such that the marginal benefit intersects the marginal cost from above. Now we show under which conditions there is at least one such intersection.

Initially, the marginal benefit is larger than the marginal cost. Consider the minimum wage $\underline{w} = 0$. Together with $b = 0$, this implies that $\bar{v} = 0$ to make the PC binding and that the equilibrium effort is 0. The marginal benefit is $\frac{E'(0)}{1} \cdot V$. Since $E'(\cdot) \equiv \frac{1}{c'(E(\cdot))}$, this is strictly positive for a minimum wage of 0. The marginal cost is $E(0) = 0$ at a minimum wage of 0. Hence, we showed that for $\underline{w} = 0$, the bonus wage's marginal benefit is higher than the marginal cost. By continuity, this also holds for some positive minimum wages.

Since the marginal benefit is initially larger, can intersect the marginal cost only from above, and both are continuous, it is sufficient to look at the limits of the minimum wage's going to infinity. Without a bonus wage, the non-compete clause will then become ever stronger which implies that the equilibrium effort will go to 1.

First, consider the marginal cost of increasing the bonus wage starting at $b = 0$. When the minimum wage goes to infinity, the equilibrium effort goes to 1 and the marginal cost goes to 1. Second, consider the marginal benefit of increasing the bonus wage starting at $b = 0.$. When the minimum wage goes to infinity, the equilibrium effort goes to 1 and the marginal benefit goes to $\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V$. Let us consider numerator and denominator separately. The numerator goes to zero because $\lim_{\underline{w}\to\infty} E'(-\bar{v}(\underline{w}, 0)) = \lim_{\underline{w}\to\infty} \frac{1}{c''(E(-\bar{v}(\underline{w},0)))}$ and $\lim_{\underline{w}\to\infty} c''(E(-\bar{v}(\underline{w}, 0))) = \infty$. This follows because $\underline{w} \to \infty$ implies that $E(-\bar{v}(\underline{w}, 0)) \to 1$ which implies that $c'(e) \to \infty$. For the same reason, the denominator also goes to zero.

Thus, we use L'Hôpital's rule to evaluate $\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V$. In order to use L'Hôpital's rule we need to check two conditions:

First, we must check that for all (positive) finite minimum wages $\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}} \neq 0$. This condition is fulfilled because $\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}} = -\frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))}$. By assumption, the numerator is positive.

Second, we must check that limit of the ratio of the derivatives exists. This condition is fulfilled because

$$\lim_{\underline{w}\to\infty} \frac{\frac{\partial E'(-\bar{v}(\underline{w},0))}{\partial \underline{w}}}{\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}}} \cdot V = \lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w}, 0)))}{[c''(E(-\bar{v}(\underline{w}, 0)))]^2} \cdot V < 1 \tag{54}$$

by assumption and continuous on (0,1).

All in all, L'Hôpital's rule yields

$$\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))}\cdot V = \lim_{\underline{w}\to\infty} \frac{\frac{\partial E'(-\bar{v}(\underline{w},0))}{\partial \underline{w}}}{\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}}}\cdot V = \lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2}\cdot V \tag{55}$$

Therefore, there is a critical minimum wage κ_4 if and only if

$$\lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} V < 1. \tag{56}$$

The assumption can also be expressed in properties of the effort cost function. It is an assumption on the convergence speeds of the second and the third derivative. Note that both $c''(\cdot)$ and $c'''(\cdot)$ go to infinity when the minimum wage goes to infinity because the equilibrium effort goes to 1 and then $c'(\cdot)$ goes to infinity. Therefore, if $(c''(\cdot))^2$ goes to infinity strictly faster than $c'''(\cdot)$, the marginal benefit converges to zero. If the convergence of $(c''(\cdot))^2$ and $c'''(\cdot)$ has the same speed, the limit is some number. If this number times V is less than 1, the assumption is also satisfied. Whenever the convergence of $c'''(\cdot)$ is faster than that of $(c''(\cdot))^2$, the assumption does not hold.

□

Having characterized which constraints bind in which combination, we can now characterize the optimal contract in each combination. Note that the contract in the first (second) combination mirrors the one in Case 1 (2). The base and bonus wages are equal and the principal does not want to use a NCC. The computations of base and bonus wage is therefore identical to the computations in Case 1 and 2 in Proposition 1 and therefore are skipped here for clarity. We now characterize the optimal bonus wage and the optimal non-compete clause depending on the the effort level that will be chosen in each combination.

Next, we consider the third combination.

Third Combination Let E be the effort level that the agent chooses given the contract. $MWC1$ binds which implies that $w = \underline{w}$. PC binds as well. We substitute IC and $MWC1$ into PC and rewrite to get

$$\bar{v} = c(E) - E\cdot c'(E) - \underline{w} \tag{57}$$

where we suppress the arguments of E and $\bar{v}$ for simplicity.

Combining $MWC1$, PC and IC by substituting for $\bar{v}$ gives

$$b = (1-E)\cdot c'(E) + c(E) - \underline{w} \tag{58}$$

Now, we substitute for w and b in P's objective function to get.

$$\pi = E\cdot V - (1-E)\cdot \underline{w} - E\cdot(1-E)\cdot c'(E) - E\cdot c(E) \tag{59}$$

P maximizes over the incentive-compatible effort level and hence $E = e_3^{NCC}$ is chosen such that

$$c(e_3^{NCC}) + (1 - e_3^{NCC}) \cdot c'(e_3^{NCC}) + e_3^{NCC} \cdot (1 - e_3^{NCC}) \cdot c''(e_3^{NCC}) = V + \underline{w}. \tag{60}$$

Next, we consider the fourth combination.

Fourth Combination Let E be the effort level that the agent chooses given the contract. $MWC1$ binds which implies that $w = \underline{w}$. $MWC2$ binds which together with the binding $MWC1$ implies that $b = 0$. $\bar{v}$ is then determined by the binding participation constraint

$$\bar{v} = -\frac{\underline{w} - c(E)}{1 - E} \tag{61}$$

The optimal effort choice is then determined by the IC and hence $E = e_4^{NCC}$ is characterized by

$$\underline{w} + e_4^{NCC} \cdot c'(e_4^{NCC}) - c(e_4^{NCC}) = c'(e_4^{NCC}) \tag{62}$$

□

A.3 Proof of Proposition 3

Proof of Proposition 3. We show that the equilibrium effort is constant in the minimum wage in the first combination, decreasing in the minimum wage in the second combination and increasing in the minimum wage if P uses a NCC, that is, in the third and fourth combination.

We start with the first combination. Note that we showed in Propositions 1 and 2 that P does not use an NCC and induces the first-best effort level in the first combination. First-best effort level is constant at e^{FB} and hence does not change in the minimum wage.

We proceed with the second combination. Note that we showed in Proposition 2 that P does not use an NCC. The equilibrium effort is hence defined by $c'(E) = b(\underline{w})$. Since the marginal cost is increasing, the equilibrium effort gets smaller if the right hand side gets smaller. Thus, we have to show that the right hand side is decreasing in the minimum wage. The binding participation constraint gives us

$$G \equiv E(b) \cdot b - c(E(b)) + \underline{w} = 0 \tag{63}$$

We use the implicit function theorem on the binding participation contraint. From now on, we will skip the argument of E for clarity. Since G is continuously differentiable, the implicit function theorem can be used to calculate the derivative of b with respect to $\underline{w}$.

$$\frac{\partial b}{\partial \underline{w}} = -\frac{\frac{\partial G}{\partial \underline{w}}}{\frac{\partial G}{\partial b}} = -\frac{1}{E} \tag{64}$$

Hence, we get that $\frac{\partial b}{\partial \underline{w}} < 0$ which then implies that the equilibrium effort decreases in the minimum wage.

We continue with the third combination, in which the optimal contract has both a bonus wage and an NCC. We, therefore, need to evaluate their combined effect on the effort. The equilibrium effort is defined by $c'(E) = b(\underline{w}) - \bar{v}(\underline{w}, b(\underline{w}))$. Since the marginal cost is increasing, the equilibrium effort gets larger if the right hand side gets larger. Thus, we need to show that the right hand side is increasing in the minimum wage. Taking the derivative with respect to the minimum wage of the right hand side yields

$$\frac{\partial b(\underline{w})}{\partial \underline{w}} - \left(\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial \underline{w}} + \frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial b(\underline{w})} \cdot \frac{\partial b(\underline{w})}{\partial \underline{w}}\right). \tag{65}$$

To show that this expression is positive, we look at its parts in turn. We already calculated the effect of a change in the minimum wage and in the bonus wage on the NCC that makes the participation constraint bind in Lemma 6. For convenience, we reproduce the result here:

$$\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} = -\frac{1}{1 - E(b - \bar{v})} \qquad \frac{\partial \bar{v}(\underline{w}, b)}{\partial b} = -\frac{E(b - \bar{v})}{1 - E(b - \bar{v})} \tag{66}$$

It remains to characterize how the optimal bonus wage changes in the minimum wage. Again, we use the implicit function theorem on the first-order condition of the expected profit maximization problem. The FOC of P's expected profit with respect to the bonus wage is

$$Z \equiv E'(b - \bar{v}) \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) \cdot (V - b) - E(b - \bar{v}) = 0 \tag{67}$$

We will from now on skip the argument of E for clarity. Before we apply the implicit function theorem to this equation to see how b changes in $\underline{w}$, we need two intermediary derivatives: $\frac{\partial^2 \bar{v}}{\partial b \partial \underline{w}}$ and $\frac{\partial^2 \bar{v}}{\partial b^2}$. And again, we can use Lemma 6, which shows that $\frac{\partial \bar{v}}{\partial b} = -\frac{E}{1-E}$.

Thus,

$$\frac{\partial^2 \bar{v}}{\partial b \partial \underline{w}} = -\frac{E' \cdot (1 - E) \cdot \frac{\partial \bar{v}}{\partial \underline{w}} + E' \cdot E \cdot \frac{\partial \bar{v}}{\partial \underline{w}}}{(1 - E)^2} = -\frac{E'}{(1 - E)^3}, \tag{68}$$

and

$$\frac{\partial^2 \bar{v}}{\partial b^2} = \frac{-E' \cdot (1 - E) \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) - E' \cdot E \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)}{(1 - E)^2} = -\frac{E'}{(1 - E)^3}. \tag{69}$$

Since Z is continuously differentiable, the implicit function theorem can be used to get the derivative of b with respect to $\underline{w}$.

$$\frac{\partial b}{\partial \underline{w}} = -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial b}} = -\frac{-E'' \cdot \frac{\partial \bar{v}}{\partial \underline{w}} \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) \cdot (V-b) - E' \cdot \frac{\partial^2 \bar{v}}{\partial b \partial \underline{w}} \cdot (V-b) + E' \cdot \frac{\partial \bar{v}}{\partial \underline{w}}}{E'' \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)^2 \cdot (V-b) - E' \cdot \frac{\partial^2 \bar{v}}{\partial b^2} \cdot (V-b) - 2E' \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)} \tag{70}$$

$$= -\frac{\left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \cdot \frac{V-b}{(1-E)^2 \cdot (c''(E))^2} - \frac{1}{(1-E) \cdot c''(E)}}{\left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \cdot \frac{V-b}{(1-E)^2 \cdot (c''(E))^2} - \frac{2}{(1-E) \cdot c''(E)}} \tag{71}$$

Since $E(\cdot) < 1$, $c''(\cdot) > 0$, $c'''(\cdot) > 0$, $b \leq V$ and concavity $\left(\frac{c'''(E)}{c''(E)} > \frac{1}{1-E}\right)$, we get that $\frac{\partial b}{\partial \underline{w}} < 0$. Hence, a higher minimum wage implies a lower bonus wage.

Let us recap what we have shown so far. On the one hand, we found that a higher minimum wage leads to a lower bonus wage which provides less incentives. On the other hand, we found that a higher minimum wage implies a more severe NCC which provides more incentives. It remains to show that the effect on the NCC is stronger than on the bonus wage. Rearranging the marginal change of the incentives in the minimum wage (65) and plugging in yields

$$-\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial \underline{w}} + \frac{\partial b(\underline{w})}{\partial \underline{w}} \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial b}\right) \tag{72}$$

$$= \frac{1}{1-E} + \frac{\partial b(\underline{w})}{\partial \underline{w}} \cdot \left(1 + \frac{E}{1-E}\right) \tag{73}$$

$$= \frac{1}{1-E} \cdot \left(1 + \frac{\partial b(\underline{w})}{\partial \underline{w}}\right). \tag{74}$$

To show that this is positive, it now suffices to show that the bracket is positive. That is, $\frac{\partial b(\underline{w})}{\partial \underline{w}} > -1$.

Consider $-\frac{\partial b}{\partial \underline{w}}$ as it is characterized in equation (71). For simplicity, let

$$x \equiv \left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \frac{V-b}{(1-E)^2 (c''(E))^2} \quad \text{and} \quad y \equiv \frac{1}{(1-E)c''(E)}. \tag{75}$$

We have that $x < 0$ and $y > 0$. It is then easy to check that $-\frac{\partial b}{\partial \underline{w}} = \frac{x-y}{x-2y} < 1$. Which was to be shown. Therefore, the equilibrium effort is increasing in the minimum wage in the third combination.

We now show that in the fourth combination the equilibrium effort is also increasing in the minimum wage. The principal does not use a bonus wage anymore. Lemma 6 shows that $\frac{\partial \bar{v}}{\partial \underline{w}} = -\frac{1}{1-E}$ where $E(-\bar{v}(\underline{w}))$ is the solution to the agent's incentive problem. This shows that higher minimum wages lead to more severe NCCs which then leads to higher effort through the incentive constraint.

To sum up, if $\underline{w} > \kappa_2$, then higher minimum wages lead to more effort incentives, and, thus a non-monotonicity of the equilibrium effort.

□

A.4 Proof of Proposition 4

Proof of Proposition 4. We show that the principal induces a higher effort level than first-best effort if the minimum wage is sufficiently large. Due to the Inada conditions, the first-best effort level will be strictly smaller than 1. We show that the equilibrium effort in the third (in case the fourth combination does not exist) and in the fourth combination must go to 1. We start with the third combination. Formally, we want to show that

$$\lim_{\underline{w}\to\infty} E(b(\underline{w}) - \bar{v}((\underline{w}), b(\underline{w})) = 1 \tag{76}$$

where E is continuous and monotonically increasing in the bonus wage, in the severity of the NCC, and in the minimum wage (Proposition 3). Therefore, we can rewrite the limit such that

$$\begin{aligned}
&\lim_{\underline{w}\to\infty} E(b(\underline{w}) - \bar{v}((\underline{w}), b(\underline{w}))) && (77)\\
&= \lim_{\underline{w}\to\infty} (c')^{-1}(b(\underline{w}) - \bar{v}((\underline{w}), b(\underline{w}))) && (78)\\
&=(c')^{-1}\left(\lim_{\underline{w}\to\infty} b(\underline{w}) - \lim_{\underline{w}\to\infty} \bar{v}((\underline{w}), b(\underline{w}))\right) && (79)\\
&=(c')^{-1}(\infty) && (80)\\
&=1 && (81)
\end{aligned}$$

due to the Inada conditions.

We proceed with the fourth combination. Formally, we want to show that

$$\lim_{\underline{w}\to\infty} E(-\bar{v}\,(\underline{w})) = 1 \tag{82}$$

where E is continuous and monotonically increasing in the severity of the NCC, and in the minimum wage (Proposition 3). Therefore, we can rewrite the limit such that

$$\begin{aligned}
&\lim_{\underline{w}\to\infty} E(-\bar{v}\,(\underline{w})) && (83)\\
&= \lim_{\underline{w}\to\infty} (c')^{-1}(-\bar{v}\,(\underline{w})) && (84)\\
&=(c')^{-1}\left(-\lim_{\underline{w}\to\infty} \bar{v}\,(\underline{w})\right) && (85)\\
&=(c')^{-1}(\infty) && (86)\\
&=1 && (87)
\end{aligned}$$

due to the Inada conditions.

□

A.5 Proof of Proposition 5

Proof of Proposition 5. Let the principal's expected profit be strictly quasi-concave in the bonus wage, that is,

$$\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{2}{E(b-\bar{v}(\underline{w},b))} \tag{88}$$

holds for all minimum wages.

With a bounded NCC we have the additional constraint that $\bar{v} \geq \underline{\bar{v}}$. This changes P's maximization problem to

$$\max_{b} \quad -\underline{w} + E(b-\bar{v}) \cdot (V-b) \tag{89}$$

$$\text{subject to} \quad \bar{v} = \max\{\bar{v}(\underline{w},b), \underline{\bar{v}}\} \tag{NCC}$$

$$b \geq b_2^{**}(\underline{w}), \tag{90}$$

where again $\bar{v}(\underline{w},b) = -\frac{\underline{w}+E(b-\bar{v}(\underline{w},b))\cdot b - c(E(b-\bar{v}(\underline{w},b)))}{1-E(b-\bar{v}(\underline{w},b))}$. The NCC condition already uses that the profit is increasing in more severe non-compete clauses (because the optimal bonus wage is smaller than the success payoff). Thus, the principal would never use a NCC that is less severe than the one NCC that makes the PC binding ($\bar{v}(\underline{w},b)$), except this would violate the bound on NCCs ($\underline{\bar{v}}$). As a result, the optimal NCC is determined by which constraint binds first: the participation constraint ($\bar{v}(\underline{w},b)$) or the bound on NCCs ($\underline{\bar{v}}$).

We now split the minimum wages into two ranges. One for which the bound on NCCs is insubstantial and one for which the bound on NCCs makes the formerly optimal contracts infeasible. This is possible because the optimal $\bar{v}$ without a bound decreases continuously and strictly monotonically in the minimum wage above κ_2. Moreover, $\bar{v}$ lies between zero and minus infinity such that any bound binds for some minimum wages. We define $\underline{w}_{bound}$ as the minimum wage for which the optimal contract without a bound on NCCs uses an NCC that is exactly the bound. That is, the optimal contract is $(\underline{w}_{bound}, b(\underline{w}_{bound}), \underline{\bar{v}})$. As argued above, $\underline{w}_{bound}$ exists and is unique for each bound $\underline{\bar{v}}$.

Case i) $\underline{w} < \underline{w}_{bound}$. For these minimum wages, the optimal contract without a bound on the NCCs does not violate the bound on the NCCs. Since the bound only introduces another constraint, these contracts remain optimal. The bound on NCCs can be ignored.

Case ii) $\underline{w} \geq \underline{w}_{bound}$. For all minimum wages above $\underline{w}_{bound}$, the optimal contracts without a bound on the NCCs are not feasible anymore: they violate the bound on NCCs. In the simplified problem, the only choice variable of the principal is the bonus wage. Thus, the optimal NCC is implicitly defined by the optimal bonus wage.

For $\underline{w} \geq \underline{w}_{bound}$, the constraint $b \geq b_2^{**}(\underline{w})$ can be ignored. The constraint said that, firstly, the participation constraint must not be violated if $\bar{v} = 0$, and, secondly, that the bonus wage must be non-negative. Since the optimal NCC at $\underline{w}_{bound}$ is strictly negative (and because of its comparative statics), we know that the participation constraint without an NCC would be satisfied. Furthermore, the optimal bonus wage can never be negative because there is a profitable deviation, as argued in the proof of Proposition 2; this deviation exists independently of a bound on the NCC.

For minimum wages $\underline{w} \geq \underline{w}_{bound}$, the optimal contract without a bound is either the one from the third combination or the one from the fourth combination. We can distinguish these as different cases. For each case, we show that once the binding NCC is optimal, it will remain optimal for all larger minimum wages, and we characterize the optimal bonus wage.

a) The optimal contract for the minimum wage $\underline{w}_{bound}$ is from the third combination. That is, the optimal bonus wage without a bound is strictly positive. Thus, the optimal bonus wage is determined by the first-order condition; the bonus wage for which the marginal profit gets zero. It is unique because the objective function is quasi-concave by assumption. For $\underline{w} = \underline{w}_{bound}$, the optimal contract remains optimal and just makes the bound on the NCC binding. Thus, the marginal profit at the bonus wage $b(\underline{w}_{bound})$ is 0. We will reconsider this particular minimum wage after describing the marginal profit in the bonus wage in general.

How does the marginal profit with respect to the bonus wage behave for a fixed minimum wage $\underline{w} > \underline{w}_{bound}$? For a sketch of the marginal profit, see Figure 7.

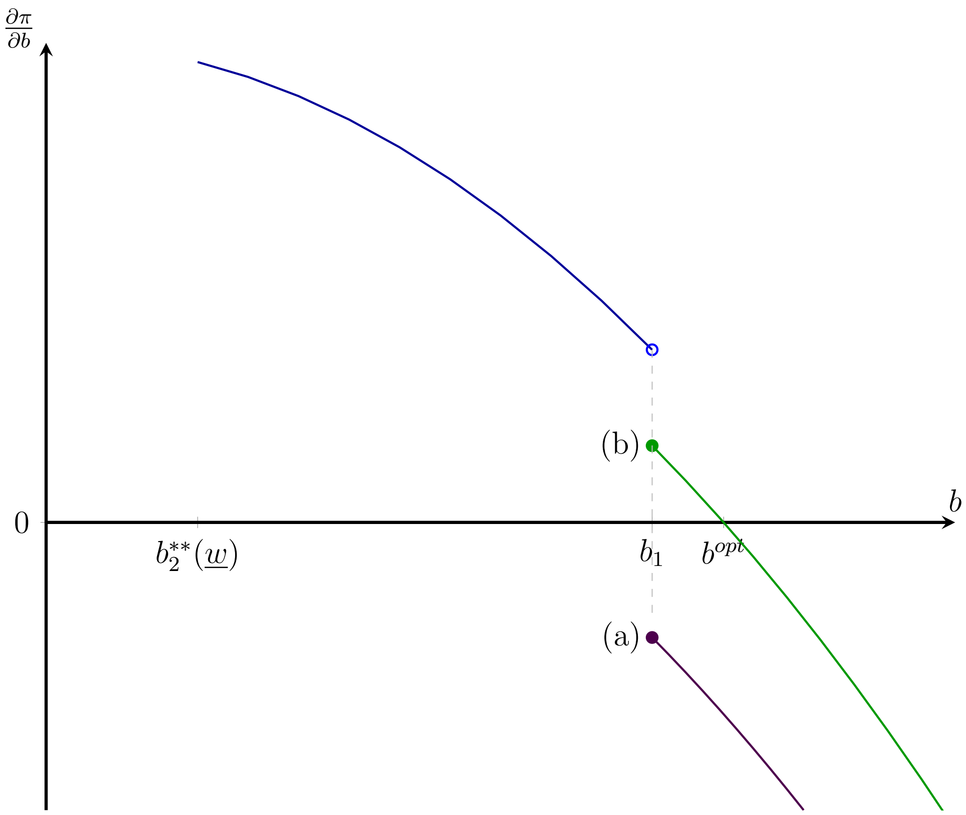

Figure 7: The Derivative of the Profit with Respect to the Bonus Wage Jumps Down as Soon as the Bound on the NCC Binds. If (a) the Jump Ends below Zero, the Agent Gets no Rent. If (b) the Jump Ends above Zero, the Agent Gets a Rent and the Optimal Bonus Wage is the Same for Higher Minimum Wages. Drawn for a Concave Objective Function.

As mentioned above, starting at $b_2^{**}(\underline{w})$, the marginal profit is positive. When increasing the bonus wage, it keeps being positive. Moreover, it has the same values as in the problem without a bound. Then, the bonus wage, $b_{bound}(\underline{w})$, is reached that allows the principal to reach the bound $\bar{v}(\underline{w}, b_{bound}(\underline{w})) \stackrel{!}{=} \underline{v}$. Importantly, at this minimum wage, the derivative is still positive: The optimal bonus wage is $b(\underline{w})$ and by the case assumption it is true that $\bar{v}(\underline{w}, b(\underline{w})) < \underline{v}$. Because $\bar{v}(\underline{w}, b)$ is decreasing in the bonus wage, and because the root of the first-order condition is unique, we know that $b_{bound}(\underline{w}) < b(\underline{w})$. From $b_{bound}(\underline{w})$ on, the principal cannot make the NCC more severe when increasing the bonus wage. Therefore, there are no double incentives anymore. The marginal profit, thus, jumps downwards to its level in the benchmark; formally

$$\lim_{b' \to b_{bound}(\underline{w})^-} \frac{\partial \pi}{\partial b}(\underline{w}, b') = \frac{E'(b_{bound}(\underline{w}) - \underline{v})}{1 - E(b_{bound}(\underline{w}) - \underline{v})} \cdot (V - b_{bound}(\underline{w})) - E(b_{bound}(\underline{w}) - \underline{v}) > 0 \quad (91)$$

$$\lim_{b' \to b_{bound}(\underline{w})^+} \frac{\partial \pi}{\partial b}(\underline{w}, b') = E'(b_{bound}(\underline{w}) - \underline{v}) \cdot (V - b_{bound}(\underline{w})) - E(b_{bound}(\underline{w}) - \underline{v}). \quad (92)$$

For bonus wages after the jump, the profit function is strictly concave in the bonus wage, as in the benchmark.[19] The marginal profit is, thus, strictly decreasing.

The optimal bonus wage is now either $b_{bound}(\underline{w})$, if the marginal profit jumps (weakly) below zero, or a higher bonus wage if the marginal profit remains positive after the jump. In any case, this implies that $\bar{v}(\underline{w}, b) \leq \underline{v}$ in the optimum. Therefore, the bound on the NCC is the binding constraint; thus $\underline{v}$ is the optimal NCC.

To find the optimal bonus wage, we have to find out which constraints will bind. This depends on whether the optimal bonus wage is at the jump point or not. If it is at the jump point, the participation constraint binds ($\bar{v}(\underline{w}, b_{bound}(\underline{w})) = \underline{v}$); which implies that the agent gets no rent. If it is to the right of the jump point, the participation constraint is slack because the NCC that would make the participation constraint binding lies outside the bound. Therefore, it is slack; which implies that the agent gets a rent.

For the other constraints (MWC_1, MWC_2, NCC), the same reasoning as above, in the proof of Proposition 2, applies. The minimum wage condition on the base wage binds. Otherwise, there is a profitable deviation. Due to the case assumption, the optimal bonus wage without a bound is positive, thus MWC_2 is slack. With a bound, it might also be that MWC_2 binds if ignoring the constraint leads to a violation. Due to the case assumption, an NCC is used, which means that the NCC feasibility constraint is slack. As a result, the optimal base wage always is the minimum wage and, as shown above, the optimal NCC is the binding NCC.

Firstly, we now determine the optimal bonus wage depending on where the jump ends and then, secondly, we show that there always is a range of minimum wages for which the jump ends in the negative.

We start with the case in which the marginal profit's jump ends in the non-positive. In this case, the optimal bonus wage is at the jump point and makes the participation constraint binding. Thus, the participation constraint pins down the optimal bonus wage. How does the optimal bonus wage change in the minimum wage? We use the implicit function theorem to show that the bonus wage that makes the participation constraint binding is strictly decreasing

[19]The second derivative is $E''(\cdot) \cdot (V - b) - 2E'(\cdot) < 0$. $E''(\cdot)$ is globally negative and $E'(\cdot)$ is globally positive.

in the minimum wage. Rearrange the binding participation constraint to

$$Z \equiv \underline{w} + E(b_{bound} - \underline{v}) \cdot (b_{bound} - \underline{v}) + \underline{v} - c(E(b_{bound} - \underline{v})) = 0. \tag{93}$$

Because this is continuously differentiable, the implicit function theorem can be used to get the derivatives of b_{bound} with respect to $\underline{w}$.

$$\frac{\partial b_{bound}(\underline{w})}{\partial \underline{w}} = -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial b_{bound}}} = -\frac{1}{E'(\cdot) \cdot (b_{bound} - \underline{v}) + E(\cdot) - c'(E(\cdot)) \cdot E'(\cdot)} \tag{94}$$

$$= -\frac{1}{E(\cdot)} \tag{95}$$

where we suppress the argument of E for clarity. The simplification is due to the agent's first-order constraint, $b_{bound} - \underline{v} - c'(E) = 0$. Since $E(b_{bound} - \underline{v}) > 0$, the bonus wage that makes the participation constraint binding is strictly decreasing in the minimum wage.

Further, we can say that the optimal bonus wage with a bound lies below the optimal bonus wage without a bound on the NCC. In both cases, the participation constraint is binding and the bonus wage is positive (due to the case assumption). Without a bound on the NCC, the optimal NCC is weakly more severe than the bound because $\underline{w} \geq \underline{w}_{bound}$; strictly more severe if $\underline{w} > \underline{w}_{bound}$. For a fixed minimum wage, a strictly more severe NCC needs a strictly larger bonus wage to keep the participation constraint satisfied. Thus, with a bound on the NCC, the optimal bonus wage is smaller.

When the optimal bonus wage hits zero, it stays at zero for all larger minimum wages. It can never become negative because of the profitable deviation. Note that when the bonus wage hits zero, for all larger minimum wages the participation constraint is slack and the agent gets a rent.

We now look at the optimal bonus wage if the jump in the marginal benefit ends in the positive and the participation constraint can be ignored. The optimal bonus wage is constant because the minimum wage does not enter the problem anymore. The optimal bonus wage is determined by the marginal profit's being zero or the minimum wage condition on the bonus wage. We define b_3 as the root.

$$\frac{\partial \pi}{\partial b} \overset{!}{=} 0 \quad \Longleftrightarrow \quad b_3: \quad E'(b_3 - \underline{v}) \cdot (V - b_3) - E(b_3 - \underline{v}) = 0 \tag{96}$$

Note that $E'(\cdot)$ is decreasing in its arguments because $E''(\cdot) < 0$. Furthermore, $E(\cdot)$ is increasing in its arguments. Therefore, compared to the third case in the benchmark, the marginal benefit of the bonus wage is smaller and the marginal cost is larger for all bonus wages. We shift $E'(\cdot)$ to the left and $E(\cdot)$ to the right. Thus, $b_3 < b^{***}$. If the marginal profit is zero for a negative bonus wage, the optimal bonus wage is zero because of the minimum wage condition. Thus, the optimal bonus wage is $b_3^+ \equiv \max\{0, b_3\}$.

What is the relation between the solution when the jump ends in the negative and when it ends in the positive? The maximization problem when ignoring the participation constraint yields a weakly larger maximum than taking into account the participation constraint. Therefore, the profit with b_3^+ is weakly larger than the profit with $b_{bound}(\underline{w})$. b_3^+ is optimal whenever it does not violate the participation constraint.

We now show that there are some minimum wages for which b_3^+ does violate the participation constraint, such that $b_{bound}(\underline{w})$ is the optimal solution. Reconsider the minimum wage $\underline{w}_{bound}$. The optimal contract is $(\underline{w}_{bound}, b(\underline{w}_{bound}), \bar{\underline{v}})$. By the case assumption, $b(\underline{w}_{bound}) > 0$. Thus, without a bound on NCCs, the marginal profit of an additional unit of bonus wage is 0 at $b(\underline{w}_{bound})$. With a bound on NCCs, this is the bonus wage at which the jump from double incentives to incentives (only through bonus wage) happens. The jump, thus, has to end in the negative. Thus, this is one minimum wage for which the participation constraint would be violated for b_3^+. Furthermore, the point at which the jump ends, moves continuously in the minimum wage: The marginal profit is a continuous function in the bonus wage and the bonus wage at which the jump happens is a continuous function of the minimum wage. Thus, the jump also ends in the negative for some larger minimum wages.

b) The optimal contract for the minimum wage $\underline{w}_{bound}$ is from the fourth combination, that is, the optimal bonus wage is 0. With a bound on the NCC, the optimal contract now is $(\underline{w}, 0, \bar{\underline{v}})$. A positive bonus wage cannot increase the profits. The optimal contract only falls into the fourth combination if the marginal profit from the first unit of bonus wage is negative. Because the binding NCC does not violate the participation constraint even without a bonus wage, there never are double incentives. Thus, the marginal profit is smaller than without a bound on the NCC (intuitively, the jump happened for a negative bonus wage). Since the marginal profit was negative with double incentives, the marginal profit is still negative. It is optimal not to use a positive bonus wage. A negative bonus wage cannot increase the profits because this means increasing the base wage above the minimum wage (otherwise the minimum wage constraint on the bonus would be violated). Then, there is a profitable deviation (making the NCC less severe, the bonus wage larger and the base wage lower by one marginal unit).

□

B Figures

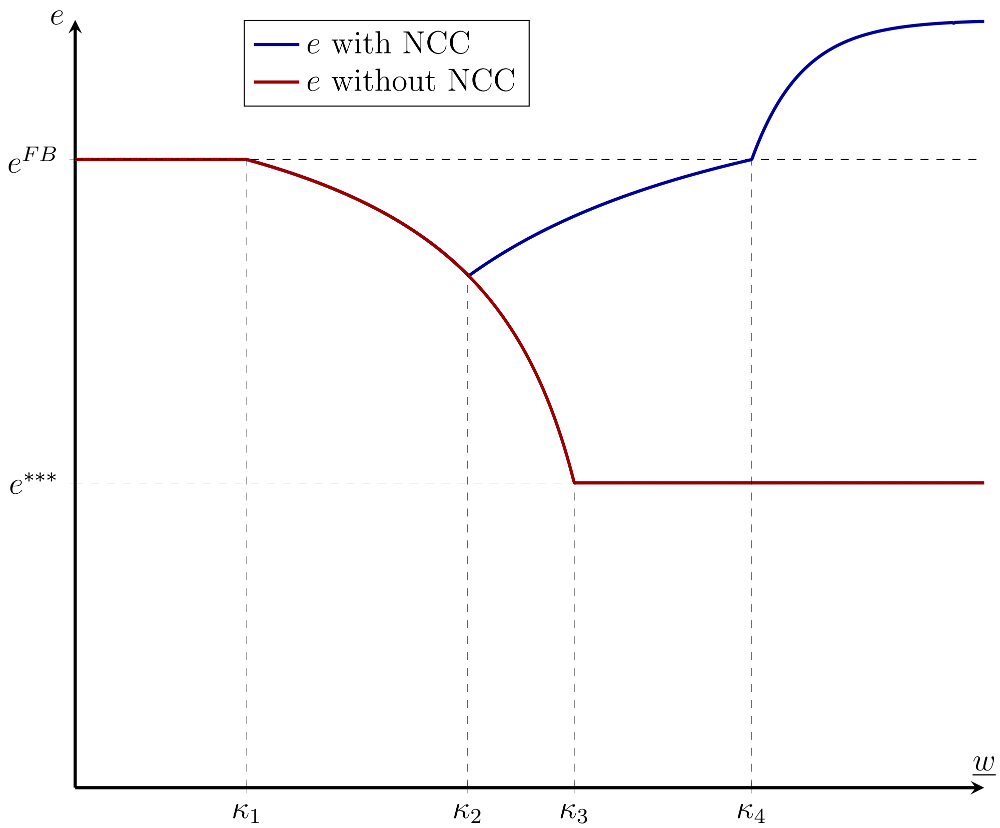

Figure 8: Comparison of the Equilibrium Efforts.

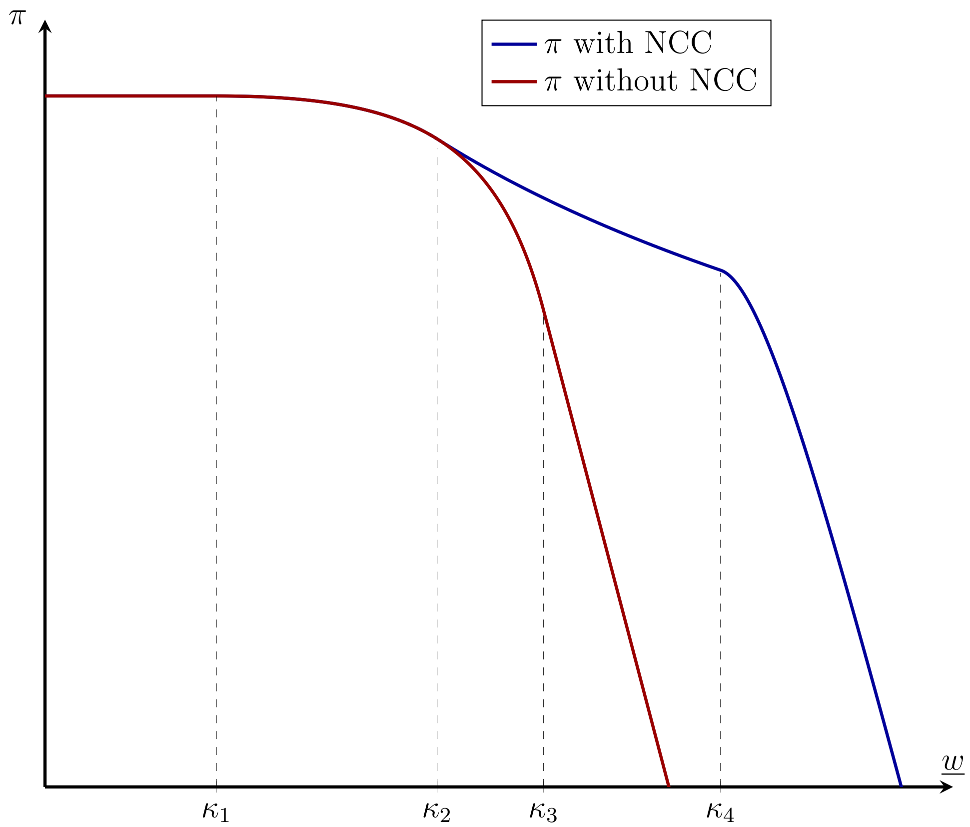

Figure 9: Comparison of Expected Profits.

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