ECONtribute

Discussion Paper

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Yuliyan Mitkov Ulrich Schüwer

June 2020

ECONtribute Discussion Paper No. 012

Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany´s Excellence Strategy – EXC 2126/1– 390838866 is gratefully acknowledged.

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Unequal and unstable: income inequality and bank risk*

Yuliyan Mitkov
University of Bonn
ymitkov@uni-bonn.de

Ulrich Schüwer
University of Bonn
schuewer@uni-bonn.de

June 15, 2020

Abstract

We document that the dispersion of failure risk across banks within a given region in the U.S. is greater in regions that have higher income inequality. We explain this pattern with a model based on risk shifting incentives where banks issue insured deposits and choose the riskiness of their portfolios. In equilibrium: (i) some banks endogenously specialize in safe lending, while others engage in risk shifting and (ii) a competition to risk shift emerges whereby loans to subprime borrowers carry negative NPVs. The dispersion of bank risk generated by this sorting is magnified in more unequal regions with greater subprime credit segments.

Keywords: Inequality, Financial stability, Agency costs, Composition of credit, Banking competition.
JEL Classification: G11, G21, G28, G51.

*We thank Martin Brown, Marie Hoerova, Todd Keister, Ralf Meisenzahl, Steven Ongena, Andre Stenzel, Martin Schmaltz, Elu von Thadden and seminar participants at the University of Bonn, the Bank of Canada, Rutgers University, the University of Mannheim and the Mainz CRC Conference for useful comments. Yuliyan Mitkov received funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC 2126/1– 390838866.

1 Introduction

The topics of income inequality and financial stability have received considerable attention over the recent years, and especially in the past decade. Nevertheless, possible mechanisms linking these two topics remain relatively unexplored. This is even more so as one goes beyond national trends and ask how the regional developments in income inequality shape the stability of the regional banking markets. In this paper we contribute to research on how income inequality shapes financial stability in two ways. First, we establish a pattern in the data between the distribution of failure risk across banks in a given region and the distribution of income in the same region. Second, we propose a model to explain this data pattern in a parsimonious way.

Empirical patterns. We begin by examining the statistical relation between the level of income inequality in a given metropolitan area in the U.S. and the distribution of the failure risk across banks that mainly operate in this region.[1] We capture the level of income inequality in a metropolitan ara with the Gini coefficient. We focus on regional banks that operate mainly within a single Metropolitan Statistical Area (MSA) in the U.S., which is the case for about 95 percent of all banks, and exclude large national banks such as Bank of America. Banks' failure risks are measured as predicted probabilities of default obtained from a probability model commonly used in the literature. Finally, we summarize the distribution of bank risk within a given MSA by taking averages, standard deviations or different percentiles of the distribution of the predicted probabilities of bank failure.

The main result of this exercise is that the dispersion of failure risk across banks is greater in regions with higher income inequality. Further, our results show that the most risky banks per MSA (top 10 percent) in these regions have a relatively higher failure risk compared with the most risky banks in regions with lower income inequality. The least risky banks per MSA (bottom 10 percent) in regions with higher income inequality tend to have a relatively lower failure risk compared with the least risky banks in regions with lower income inequality, but these relations are largely statistically insignificant. Finally,

[1]For the demarcation of a metropolitan area, we use the definitions of Metropolitan Statistical Areas (MSAs) by the U.S. Office of Management and Budget.

our evidence suggests that the average failure risk of banks per MSA does not depend on inequality.

Keeley's observation and the need for more analysis. What can account for the patterns in the data? Why do we observe a positive association between the dispersion of bank risk and income inequality, but not between income inequality and the mean level of bank risk? To begin, more unequal regions in the US might be characterized by larger shares of high-risk low-income households. At first glance, it might seem obvious that regions with higher income inequality would have higher risk of bank failure because borrowers on average are riskier. However, banks' lending decisions do not passively follow the risk of their potential borrowers, but instead are endogenously determined as pointed out by Keeley (1990):

> There is little doubt that increased risk in the economy and declining capital ratios have a lot to do with increased bank risk. But these developments do not explain why banks allow bankruptcy risk to increase. After all, depository institutions have considerable control over the riskiness of their asset portfolios and perhaps even more control over their capital ratios.

Hence, it is not clear why the observed relations between inequality and bank risk exist, and which mechanisms are at play. This is even more so if equilibrium effects are considered.

Model preview. We propose a stylized model to explain how these patterns can emerge in equilibrium. In the model, ex-ante identical banks issue insured deposits and select the riskiness of their portfolios of loans. Ex-ante, each bank decides whether to specialize in risk shifting practices or in safer lending. Risk shifting is the only friction in this setup and it can be sustained because of deposit insurance. The model is best interpreted as describing the banking market equilibrium within a single region.

One can think of risk shifting is a real option available to each bank which is exercised if the bank starts lending to high-risk borrowers without adequate safety provisions in terms of screening, LTVs and so on. However, we depart from the partial equilibrium logic of most models based on risk shifting by imposing the equilibrium requirement that banks engaged in risk shifting are not more profitable (in expectation) than banks who do not engage in this activity. Specifically, competition in the loan market implies that loan terms to each type of household adjust to ensure that all banks have the same expected profits - a process which we term the *competition to risk shift*.[2]

We embed the competition to risk shift mechanism into a stylized model of mortgage credit based on the home-equity approach to mortgage default. In this setup, mortgage default for a given borrower takes place when the negative equity in the house - the amount with which the face value of the loan exceeds the value of the house - exceeds this borrower's default cost. The magnitude of the default cost reflects stigma effects, transaction costs etc. Conditional on having negative equity, we assume that the probability of default is higher for low-income households.[3] In equilibrium, low-income households are more likely to default on their mortgage, and therefore, carry greater credit risk.[4]

Preview of the model results. To fix ideas - and to isolate the source of bank risk in this framework - we examine first the case in which risk shifting is prohibitively costly due to regulation, reputation considerations, or loss of franchise value. In this case, the equilibrium is relatively straightforward - each bank has a zero probability of failure. Banks ensure that they always remain solvent by holding sufficient capital or by limiting their exposure to high-risk borrowers. Thus, the distribution of income will not shape the distribution of bank risk in this environment unless there are agency costs. This result reflects the argument by Keeley that banks can choose their risk level independently from the risk in the economy.

Now suppose risk shifting is not prohibitively costly (e.g. because of low bank franchise values). We label a borrower as being part of the high-risk credit segment if and only if

[2] This mechanism appears in different guises in the literature on rational bubbles based on risk shifting and limited liability (see, e.g., Allen and Gorton, 1993; Allen and Gale, 2000). We draw more parallels to the existing literature in the next section.

[3] Specifically, we assume that households with lower income have a higher cost of funds. For instance, households that need to borrow on their credit card to meet their mortgage payment have a cost of funds equal to the interest rate on the credit card. On the other hand, households that can repay the mortgage out of their savings have a cost of funds equal to the risk-free interest rate (see, e.g. Foote et al., 2008, and the references therein).

[4] In the baseline version of the model we abstract from housing market speculation to streamline the exposition. Speculative borrowers (also called real estate investors) have an incentive to maximize leverage in order to benefit from capital gains and are also more likely to obtain non-standard mortgages, such as Adjustable Rate Mortgages which are intrinsically riskier than standard fixed rate mortgage. See Haughwout et al. (2011) for more details. Once we have presented the main mechanism it will become clear that speculative credit demand would amplify the effect of inequality on bank risk.

he can obtain equilibrium credit from banks specialized in risk shifting. Otherwise, this borrower belongs to the low-low-risk segment. The equilibrium outcome is characterized by an endogenously determined threshold income level such that all households with income below the threshold are part of the high-risk credit segment whereas all households with income above the threshold are part of the low-risk credit segment.

If the equilibrium threshold separating high from low risk households is located at an interior point of the income distribution then two types of banks emerge – safe and risky. Safe banks do not engage in risk shifting, lend only to prime borrowers, and always remain solvent. Risky banks, on the other hand, lend only to subprime borrowers, engage in risk shifting, and become insolvent with positive probability.

The separation of ex-ante identical banks into safe and risky emerges because the competition among banks looking to risk shift pushes down the loan rates to high-credit risk borrowers to a point where these loans carry negative net present value to the lender. As a result, high-risk borrowers will be able to borrow only from banks specializing in risk shifting. The remaining banks strictly prefer to hold storage (i.e. the safe asset in our model) or to lend to low-risk borrowers. [5]

The equilibrium fraction of banks specializing in risk shifting adjusts to satisfy the demand for credit originating in the high-risk credit segments. Simultaneously, the equilibrium fraction of safe banks adjusts to satisfy the demand for credit originating in the low-risk credit segments. In other words, safe and risky banks co-exist and focus on different types of clienteles. Furthermore, the equilibrium sorting of banks into safe and risky will be more pronounced (and the dispersion of bank risk greater) in regions with more unequal distribution of income. At the same time, the effect on the mean level of bank risk is ambiguous and it depends on the relative size of the high-risk and the low-risk credit segments.

We can thus account for the patterns in the data parsimoniously by augmenting the standard franchise value hypothesis (see, e.g., Keeley, 1990) with the *competition to risk shift hypothesis* proposed in this paper.[6]

[5] The fact that high-interest loans carrying negative NPV can nevertheless remain attractive for banks specialized in risk shifting is well understood. What is new in this setup is that negative NPV loans emerge endogenously for households with income below a certain threshold which, in turn, drives ex-ante identical banks to follow very different lending strategies and to face very different risk of failure in equilibrium.

[6] While the patterns in the data are consistent with the *competition to risk shift* hypothesis, it must

Note that the competition to risk shift and the sorting of banks it implies will not take place unless banks' franchise values are sufficiently low (e.g. due to low entry barriers in regional markets). At the same time, the fact that some of the banks within a region behave prudently and do not take excessive risks does not necessarily imply that they have large franchise values - even when all banks have equally low franchise values our model predicts that some of them specialize in prudent behavior.[7]

The paper proceeds as follows. We review the related literature in Section 2. Section 3 presents the empirical patterns. Section 4 presents the model. Section 5 applies the model to study how the distribution of income shapes bank risk. Section 6 concludes. All figure and proofs are collected in the appendix.

2 Related literature

This paper is related to the literature examining the connection between inequality and financial instability. Rajan (2010) argues that rising inequality in the past three decades led to political pressure for redistribution that eventually materialized in the form of subsidized credit to low income segments of the population undermining financial stability in the process. Kumhof et al. (2015) develop a model in which financial crises are triggered by increased income share going to the high-income households. The reason is that the high-income households accumulate part of their wealth in loans to low-income households which leads the later to become more indebted and the financial system more fragile.

In our model, the competition to risk shift ends up promoting easy credit and high leverage among the low-income segments of the population. At the same time, the underlying reason for financial instability is distinct from the political motivations to redistribute as in Rajan (2010) or wealth accumulation preferences of the high-income agents as in Kumhof et al. (2015). Furthermore, to the best of our knowledge, none of the ex-

be highlighted that we stop short of a direct test of this theory which requires *(i)* identifying the negative NPV loans and then *(ii)* showing that these loans are concentrated in a subset of the banks. This is especially challenging since in step *(i)* we need to establish that the bank *knew* that the loan was NPV at the time it was made.

[7]Note that the sorting of banks into safe and risky implies that some of the banks choose to hold undiversified portfolios of high-risk loans without taking adequate provisions in terms of risk management. As a result, the capacity of the regional banking system to absorb economic shocks could be undermined and the risk sharing possibilities between different banks (both within and across regions) could remain not fully exploited.

isting studies on the relation between inequality and financial instability is motivated by the particular patterns we find in the data.

The implications of risk-taking incentives for banks' portfolios have been examined by Rochet (1992) and Repullo and Suarez (2004) who show that deposit insurance may push banks to endogenously specialize into different risk categories. Harris et al. (2018) show that risk-taking incentives together with heterogeneous borrowers can generate bank specialization and lead to distortion in the pricing of loans. Our analysis compliments this literature by applying what we term the competition to risk-shift mechanism to explain how the distribution of income can shape bank risk.

The competition to risk-shift mechanism is related to the model of rational bubbles in Allen and Gale (2000). In their model, the possibility of risk shifting leads investors protected by limited liability to bid up the price of risky assets above fundamentals because they can avoid losses in low payoff states by defaulting.[8] In our model rational bubble can be seen as taking the form of below break-even interest rates on high-risk borrowers.

The paper is also related to the theoretical literature on the effect of banking competition on financial stability. One set of theories predicts a positive link between competition and bank risk-taking. Specifically, competition may decrease bank franchise value and weaken the incentive of banks to behave prudently (Keeley, 1990; Allen and Gale, 2000; Corbae and Levine, 2019). In contrast, Boyd and De Nicolo (2005) present a model with adverse selection in which more competition can lead to reduced bank risk-taking. In particular, if bank competition reduces the interest on loans, then the incentive of bank borrowers to take on riskier projects is weaker and loans become safer. Martinez-Miera and Repullo (2010) integrate both factors and show that the effect of competition on bank risk-taking can be non-linear and vary from one economy to another.

We abstract from adverse selection issues (which is arguably justified given that we focus on mortgage markets and consumer credit) and present a model in which more competition makes some of the banks riskier while the other safer. The net effect on the overall level of bank risk is therefore ambiguous and will depend on the local economic conditions, including the income distribution. This is consistent with the mixed empirical

[8] Similarly, in Allen and Gorton (1993) stock prices can exceed fundamentals (i.e. a rational bubble can occur) because portfolio managers have an incentive to trade not based on information or liquidity needs but rather with the intention of profiting at the expense of the investors whose money they manage.

evidence in the literature with some studies finding that competition leads to more bank risk-taking (see, e.g., Jiménez et al., 2013; Braggion et al., 2017; Gissler et al., 2019), while others finding the opposite effect (see,e.g., Jayaratne and Strahan, 1998; Schaeck et al., 2009; Carlson and Mitchener, 2009).

Finally, the paper is related to the literature documenting that credit supply shocks caused either by perverse securitization incentives or misguided expectations can lead to inflated house prices and trigger financial distress (see, e.g., Mian and Sufi, 2009; Favara and Imbs, 2015; Adelino et al., 2016). A related strand of the literature has examined the relation between income inequality and household debt and has shown that it can exacerbate economic fluctuations (Krueger and Perri, 2006; Iacoviello, 2008; Midrigan and Philippon, 2011). In our model, the lending rates prevailing in high-risk credit segments can fail to properly capture borrowers' credit risk and, in fact, may reflect subsidized credit through negative NPV loans. We thus complement this literature by highlighting an additional channel that can contribute to financial instability.[9]

3 Empirical patterns

This section explores patterns between measures of income inequality and bank risk. To the best of our knowledge, the existing literature has not looked into this relation. The goal is to document relevant correlations, not necessarily causal relations. The findings from this section serve as the empirical motivation and foundation for the theoretical model in this paper.

Level of Analysis. We are interested in the cross-sectional variation of inequality and bank risk across different regions. In principle, we could consider the data across different countries, across different metropolitan areas, across different counties, or across other geographic boundaries. We decided to use Metropolitan Statistical Areas (MSA) as defined by the U.S. Office of Management and Budget as geographic boundary. An MSA is a geographical region with a relatively high population density at its core and close eco-

[9]As in Arellano (2008), default in our model is rational and more likely in recessions (which tends to be accompanied by drop in house prices). Rational default provides relief for low-income households but at the same time is accompanied by collapse in real economic activity which might hit low-income households especially hard.

nomic ties throughout the area. An example is the *Washington–Arlington–Alexandria, DC–VA–MD–WV* metropolitan statistical area. A total of 330 MSAs existed across the USA in the year 2000. Data on the economy and inequality from the Bureau of Economic Analysis is frequently available for an MSA. An MSA is also a relatively good proxy for a banking market. In particular, our data shows that a large fraction of banks operate most of their branches within their MSA, most of their deposits come from branches within the MSA, and most of their mortgages are also provided to borrowers within the MSA (see Figure 3 for the regional concentration of of branches and deposits). We exclude national banks such as Bank of America from the analysis. Further, the definition of a banking market that is used by the banking regulator in bank merger assessments is often identical or similar to an MSA.

First illustrative evidence. As illustrated in Figure 1 and Figure 2, both the level of income inequality (measured as Gini coefficients) and the average bank risk (measured as predicted probabilities of default) vary across regions in the United states. This variation allows us to explore relations between both measures. In particular, the evidence provided in this section shows the following: *(i)* Higher income inequality is associated with higher bank risk of the most risky banks per MSA and *(ii)* Higher income inequality is associated with higher dispersion of bank risk per MSA.

The following paragraphs describe in detail the sample, variables, and the analysis that leads to this evidence.

3.1 Sample and variables description

The sample includes annual data for all banks with their headquarters in a Metropolitan Statistical Area (MSA) in the U.S. and a regional focus on this area. In particular, we exclude large national banks that are not focussed on a specific region (e.g., Bank of America) by restricting the sample to banks that hold 50 percent or more of their deposits in the MSA where they have their headquarters, which is the case for about 95 percent of all banks.[10] The sample period is 2000 to 2018. Banks' financial data comes from call reports, as provided by the FDIC. Data on income inequality and other economic data

[10] In robustness test, we also use alternative criteria to exclude large national banks.

for each MSA comes from the U.S. Census Bureau and is based on American Community Survey data. An overview on all variables is given in Table 1. We require that MSA-level data on the Gini coefficient, the mean household income, the median household income as well as bank-level data on the predicted prob- ability of default is available. Finally, we only consider MSAs with at least two bank headquarters. This leads to a sample of 5,453 banks that are located in 325 MSAs across the U.S. A more detailed description of the variables is provided in the Table 1.

Income inequality. We use the Gini coefficient as a measure of income inequality. A Gini coefficient of 1 indicates perfect inequality, i.e., one household having all the income and rest having none. A Gini coefficient of 0 indicates perfect equality, i.e., all households having an equal share of income. Importantly for our analysis, there exists a significant variation in the Gini coefficient across MSAs in the U.S., as illustrated in Figure 1.

Bank risk and dispersion of bank risk. Our measures of bank risk are based on banks' predicted probabilities of default that we predict with a linear probability model. The sample of this regression includes yearly financial data of all banks with their headquarters in the contiguous United States over the period 2000 to 2018. The number of bank failures for this sample is 571, which includes both final bank failures (e.g., Washington Mutual Bank) and assistance transactions (e.g., Bank of America and Citigroup). The dependent variable of the model is a binary variable with a value of one if a bank fails in a certain year, and zero otherwise. In line with the literature, we choose the first lag of several explanatory variables that are associated with bank risk: age, cost-to-income ratio, commercial and industrial loan ratio, equity ratio, foreclosure ratio, income earned and not collected on loans, liquidity, gross loan ratio, non-performing assets ratio, real estate loan ratio, return on assets, and the natural logarithm of total assets.[11] We then calculate several measures of bank risk on the MSA level:

- The variable *P90_PD* captures the bank risk of the most risky banks per MSA. It is calculated as the long-term average per MSA of the 90th-percentile of banks' predicted probabilities of default for each MSA and year.

[11] See the Online Appendix for details and regression results of the model.

- The variable *P10_PD* captures the bank risk of the least risky banks per MSA. It is calculated as the long-term average per MSA of the 10th-percentile of banks' predicted probabilities of default for each MSA and year.

- The variable *Mean_PD* captures the average bank risk per MSA. it is calculated as the long-term average per MSA of the mean of banks' predicted probabilities of default for each MSA and year.

- The variable *SD_PD*is the long-term average per MSA of the standard deviation of banks' predicted probabilities of default for each MSA and year. We use this measure as a proxy for the dispersion of bank risk.

3.2 Summary statistics and correlations

Summary statistics are shown in Table 2. Correlations between the main variables of interest are shown in Table 3. A first graphical inspection of the relations between different measures of bank risk and income inequality is shown in Figure 3. The graphs reveal no clear relations, but point to a positive relation in the upper left panel (Gini and the most risky banks per MSA), a negative relation in the upper right panel (Gini and the least risky banks per MSA), no relation in the lower left panel (Gini and the average bank risk per MSA), and a positive relation in the lower right panel (Gini and the dispersion of bank risk per MSA).

3.3 Analysis

An ideal experiment to explore the causal relation between income inequality and bank risk is unfortunately not available. This would require a random exogenous shock on income inequality, which does not simultaneously affect bank risk. Hence, we primarily identify correlations, not causal relations.

Our analysis uses a simple OLS specifications with data from MSA j in state s:

$$PD_j = \alpha + \beta_1 Gini_j + \epsilon_j$$

where PD_j represents different measures of bank risk per MSA j. In some specifications, we also use state fixed effects, and we also include the mean household income (or alternatively the median household income) of MSA j to control for different levels of income.

Inequality and bank risk of the most risky banks per MSA. Regression results in Column (1) of Table 4 show a significantly positive relation between the Gini coefficient and the bank risk of the most risky banks, measured as the long-term average of the 90th-percentile of banks' predicted probabilities of default, per MSA. The coefficient of 0.0667 means, for example, that an MSA a with relatively high Gini of 0.48 (90th percentile) is associated with a 0.0047 (0.47 percentage points) higher probability of default of its most risky banks compared with an MSA with a relatively low Gini of 0.41 (10th percentile). Note that the average probability of default of the most risky banks over the sample period is 0.0126 (1.26 percent). Hence, the effect is economically significant.

The regression results in Columns (2) to (6) of Table 4 are qualitatively similar, with the exception of the specification in Column (4), where state fixed effects are included and the coefficient is not significantly different from zero.

Inequality and bank risk of the least risky banks per MSA. Regression results in Table 5 show largely no significantly relation between the Gini coefficient and the bank risk of the least risky banks, measured as the long-term average of the 10th-percentile of banks' predicted probabilities of default, per MSA. However, regression results in Columns (2) and (6) point towards a negative relation at a significance level of 10 percent.

Inequality and average bank risk per MSA. Regression results in Table 6 show no significant relations between the Gini coefficient and the average bank risk, measured as the long-term average of the mean of banks' predicted probabilities of default, per MSA.

Inequality and dispersion of bank risk on the MSA-year level. Regression results in Table 7 show significantly positive relations between the Gini coefficient and the dispersion of bank risk, measured as the long-term average per MSA of the standard deviation of banks' predicted probabilities of default for each MSA and year. Results are highly significant for all specifications.

Overall, the main message from this empirical exercise is that income inequality and bank risk are indeed statistically related. We find robust evidence that the dispersion of failure risk across banks is greater in regions that have higher income inequality.

4 The model

In this section, we propose a model to explain the empirical patterns presented in Section 3. Although highly stylized, the framework developed here provides a useful tool to organize our intuition about the possible channels linking income inequality and bank risk. In addition, the model is of independent interest since it demonstrates how the competition to risk-shift logic which was proposed in a different context by Allen and Gale (2000) naturally applies to the banking sector.[12]

4.1 Households

There are two dates: the initial date (date-0) and the final date (date-1) and a continuum of households. All households are risk neutral. The distribution of income at the initial date is characterized by the cumulative distribution function H over the interval $\left[\underline{y}, \overline{y}\right]$ with a p.d.f. h. Since we want to focus on the effect of inequality, the mean level of income is normalized to one $\int_{\underline{y}}^{\overline{y}} yh(y)dy = 1$.

To buy a house, households must obtain a mortgage loan. The mortgage term consist of a single period, hence full repayment is required at date-1. The outstanding balance on the mortgage is $(1 + r)P_0$, where P_0 is the house price and r is the interest rate on the mortgage (which depends on the borrower's credit risk). We assume that all households who choose to become homeowners buy one unit of standardized housing regardless of their income level. We also abstract from down-payments. Hence, the size of the loan is equal to the price of the house P_0. Both of these assumptions can be relaxed without altering the main results.

The aggregate state of the economy is realized at date-1 and is either *good* or *bad*. The probability of the bad aggregate state is $q \in [0, 1]$. The house price at date-1 is a function

[12] A complementary paper is that by Harris et al. (2018) who apply a similar logic to study equilibrium bank risk-taking. However, their approach is not motivated by the particular empirical relations we documented in Section 3.

of the aggregate state. Specifically, if the aggregate state is good, then the date-1 house price is $P_1 = hP_0$ where $h > 1$. On the other hand, if the aggregate state is bad, then the date-1 house price is $P_1 = lP_0$ where $l < 1$.

Mortgage loans are granted at date-0 and repaid at date-1 after the realization of the aggregate state. Each borrower has the option of defaulting on his mortgage. The benefit of default is that it allows for the outstanding liability on the mortgage to be canceled. The cost of default is that it triggers foreclosure whereby the lender seizes the house.

For simplicity, we assume that mortgage default potentially occurs only in the bad aggregate state. Conditional on default, the borrower incurs a default cost. The cost of default $C^{i,y}$ for borrower i with income y is drawn from the distribution $G(.|y)$ with non-negative support.[13] The ex-post optimal decision of whether to default or repay is characterized by

$$lP_0 - (1+r)P_0 \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} C^{i,y} \qquad \text{then} \qquad \left\{ \begin{array}{c} \text{repay} \\ \text{repay, default} \\ \text{default} \end{array} \right\} \tag{1}$$

where lP_0 is house price in the bad aggregate and $(1+r)P_0$ is the outstanding balance on the mortgage. Thus, default occurs when the negative equity in the house $lP_0 - (1+r)P_0$ exceeds the magnitude of the default cost $C^{i,y}$.[14] Thus, negative equity is a necessary but not sufficient condition for default.[15]

Those borrowers which are more likely to draw high cost of default are also less likely to default on their loan. The probability of default in the bad aggregate state for a borrower with income y is given by $G\left(P_0[1+r-l] \,\middle|\, y\right)$. For simplicity we assume $G(.|y) \sim U[0,\, \alpha y]$ where $\alpha > 0$ is a parameter governing the sensitivity of the default rate to the negative equity in the house. Hence, the fraction of borrowers with income y who default on their mortgage in the bad aggregate state is equal to

[13] The default cost is equal to stigma effects plus transaction costs minus the cost of funds which the borrower incurs in repaying the loan.

[14] Without loss of generality, we assumed that when indifferent, each borrower chooses to repay.

[15] Note that borrowers with positive equity in the house (i.e. the house price exceeds the outstanding obligation on the mortgage) always repay their loan. The reason is that they can avoid the default cost by selling the house and repaying the loan.

$$\frac{1}{\alpha y} P_0[1 + r - l] \tag{2}$$

The rate of default in the bad aggregate state is increasing in the negative equity in the house. In addition, for the same level of negative equity, a higher-income household is less likely to default than a lower income household. Since borrowers with higher income are less likely to default on their mortgage, the equilibrium interest rate $r(y)$ will be a *decreasing* function of the borrower's income.

4.2 Banks

There is a continuum of ex-ante identical banks each with fixed capital level $k \in (0, 1)$. We impose a minimum capital ratio: each bank can borrow at most $1 - k$. Thus, the maximum size of each bank's balance sheet is normalized to one.

Suppose a given bank borrows b and allocates its funds $b + k$ between mortgage loans and storage. Let s denote the amount held in storage and $\alpha(y)$ the amount invested in mortgage loans to households with income y. Since there is no default conditional on the good state, each \$1 invested in mortgage loans to borrowers with income y yields a return of $1 + r(y)$ in the good aggregate state. The bank's payoff in the good aggregate state is

$$\psi_G(s, \alpha) \equiv s\gamma + \int_{\underline{y}}^{\overline{y}} \alpha(y) \left(1 + r(y)\right) dy \tag{3}$$

where $\gamma \geq 1$ is the return on storage. On the other hand, the bank's payoff in the bad aggregate state is

$$\psi_B(s, \alpha) \equiv s\gamma + \int_{\underline{y}}^{\overline{y}} \alpha(y) \left[(1 + r) - G\left[P_0(1 + r(y) - l) \,\middle|\, y \right] (1 + r - l) \right] dy \tag{4}$$

In the bad aggregate state, fraction $G\left[P_0(1 + r(y) - l) \,\middle|\, y \right]$ of the borrowers with income y choose to default. Conditional on default, the bank forecloses on the house and recovers only \$$l$ on the initial \$1 invested.[16]

[16] We assume that the bank does not incur foreclosure cost. Adding these to the analysis will complicate the notation without delivering new insights.

Before the aggregate state is realized, the expected net payoff for the bank is

$$(1-q)\max\{\psi_G(s,\alpha)-(1+r_d)b,\, 0\}+q\max\{\psi_B(s,\alpha)-(1+r_d)b,\, 0\}-qF-k \tag{5}$$

where r_D is the interest rate offered to the depositors and F is the (exogenous) bankruptcy cost which is borne by the bank's owners conditional on default.[17]

Deposits are insured, which implies that the equilibrium interest required by the depositors does not very with the portfolio composition of their bank. From now on, and unless otherwise stated, the return on deposit is set to equal the return on storage $1+r_d=\gamma$. Further, the return to storage is normalized to one.

The objective of the bank is to choose its portfolio composition $\{s,\alpha\}$ and leverage ratio $b\in[0,\, 1-k]$ to maximize (5) subject to the budget constraint

$$s\gamma+\int_{\underline{y}}^{\overline{y}}\alpha(y)dy\leq k+b \tag{6}$$

In addition, there is *free-entry* which implies that the equilibrium profile of interest rates $\{r^*(y)\}_{\underline{y}}^{\overline{y}}$ would adjust to ensure that all banks earn the normal rate of return.

4.3 Supply and demand for housing

We assume that each household with income-y demands one unit of housing with probability $\eta(y)\in[0,1]$, which is non-decreasing in y. That is, the demand for housing is inelastic in the house price and the interest rate.[18] The aggregate demand for housing stock is equal to

$$D\equiv\int_{\underline{y}}^{\overline{y}}\eta(y)h(y)dy \tag{7}$$

where $h(y)$ is the p.d.f. of the income distribution.

The cost to produce one additional unit of housing is proportional to the aggregate

[17]For example, F could represent the bank's franchise value or reputational cost triggered by default. Clearly, higher F increases the incentive for prudent behavior.

[18]This assumption considerably simplifies the analysis and allows for sharper characterization of the results. The main results continue to hold as long as the demand for housing remains relatively price in-elastic.

housing stock, $c_0 + c_1 N$, where N is the aggregate stock of housing. Thus, expanding the housing stock becomes more and more expensive as the demand for housing increases.[19]

The technology to produce new housing units is operated by competitive firms, which implies that the equilibrium house price equals

$$\begin{aligned} P_0^* &= c_0 + c_1 N \\ &= c_0 + c_1 \int_{\underline{y}}^{\overline{y}} \eta(y) h(y) dy \end{aligned} \tag{8}$$

where the second line uses the equilibrium condition that the demand for housing stock must equal the supply of housing stock. Thus, the distribution of income H affects the house price P_0 through its effect on the home-ownership rate in (7).

4.4 Sequence of events

At date-0: banks issue deposits and choose their portfolio compositions. At date-1: *(i)* the aggregate state is realized. *(ii)* Each household's idiosyncratic default cost is realized. *(iii)* Each household chooses whether to default or repay on their mortgage. *(iv)* After collecting payment from all households choosing to repay and foreclosing on households choosing to default each bank is either solvent or insolvent. If the bank is insolvent it defaults. *(v)* All depositors with losses are covered by the deposit insurance agency.[20]

5 Equilibrium analysis

The analysis proceeds as follows. *First*, we define a reference set of interest rates which are used to characterize the equilibrium outcome. *Second*, we derive the equilibrium under risk shifting. *Third*, we analyze how the risk shifting incentives shape the relation between the distribution of income and bank risk. *Forth*, we connect the predictions of the model to the empirical patterns documented in Section 3.

[19]This could arise due to limited supply of land due to geographical constraints such as the presence of water or steep slopes. Another reason may be regulatory restrictions such as zoning restrictions and new housing approval rates.

[20]We assume that the deposit insurance is funded by lump-sum taxation and, for simplicity, do not model this part explicitly.

5.1 Reference interest rates

The *break-even interest rate* for a borrower with income y implies that \$1 invested in mortgage loans to income y borrowers yields \$1 in expectation. That is, $r^{be}(y)$ satisfies

$$\left(1 + r^{be}(y)\right) - qG\left[P_0\left(1 + r^{be}(y) - l\right) \middle| y\right]\left(1 + r^{be}(y) - l\right) = 1 \tag{9}$$

In other words, $r^{be}(y)$ ensures that a loan to a borrower with income level y carries zero net present value to the bank. Since the rate of default in the bad aggregate state is inversely related to the income of the borrower, we have

$$\frac{\partial r^{be}(y)}{\partial y} < 0 \tag{10}$$

Figure 5(a) depicts $r^{be}(y)$ as a function of y.

The break-even rates represent a useful reference point since they emerge in equilibrium if bankers invest their own money or if they cannot shift risk. However, if none of these conditions is satisfied, then the equilibrium interest rate profile would in general be different as we show next.

5.2 The competition to risk shift leads to sorting

The following example illustrates the choices faced by each bank. Suppose a given bank borrows $b \in [0,\, 1-k]$ in the deposit market. If the bank is unable to repay the depositors in full, then they can claim whatever the bank has. Any loss to the depositors is then covered by the deposit insurance fund. Because of deposit insurance, the return offered to the depositors is not contingent on the risk taken by the bank and normalized to unity.

First, suppose the bank combines b with its own capital of k and invests an amount 1 in storage. The payoff from this *safe strategy* is

$$1 - b - k$$

where the return to storage is normalized to one. On the other hand, suppose the bank invests 1 in risky loans. Suppose the payoff from these loans exceeds b, the amount the bank has borrowed, only conditional on the good aggregate state. Then, in the good aggregate state, the bank repays b to its depositors and makes a profit of $1 + r - b - k$, where r is the interest rate on the loans. In the bad aggregate state, the entire payoff goes to the depositors and the bank gets 0. The payoff from this *risky strategy* is

$$(1 - q)(1 + r - b) - k - qF$$

where F is the bankruptcy cost which is borne by the bank's owners conditional on default.

In equilibrium, the bank must be indifferent between investing in storage and investing in risky loans. This will be the case if and only if the payoff from the safe strategy equals the expected payoff from the risky strategy.

$$(1 - q)(1 + r^* - b) - k - qF = 1 - b - k$$

To ensure that the above is the case, the equilibrium interest rate on risky loans would adjust so that the expected payoff from storage equals the expected payoff from investing in risky loans. That is,

$$1 + r^* = \frac{1 - q(b - F)}{1 - q}$$

Note that if $r^* < r^{be}$, the loan is issued at below break-even interest rate, and therefore, carries negative net present value.

The next two results apply the above logic to derive the joint equilibrium in the mortgage and in the banking market. First, we characterize the equilibrium on the mortgage market.

Proposition 1. *(Risk shifting region) Suppose that risk-shifting is possible. Then, in any equilibrium, there is an endogenous cutoff y^* such that*

(i) $r^*(y) < r^{be}(y)$ for $y < y^*$ and

(ii) $r^*(y) = r^{be}(y)$ for $y \geq y^*$.

All loans issued to borrowers with income below y^* are priced below the corresponding break-even interest rate, and therefore, carry a negative net present value to the lender. Why would any bank grant a loan at below break-even interest rate? The answer is because of limited liability, the bank only cares about its payoff in the good aggregate state. The determination of the cutoff point y^* is depicted on Figure 5(a) and can be described as follows. First, derive the break-even interest rate $r^{be}(y)$ for each level of income $y \in [\underline{y}, \overline{y}]$. Second, the cutoff point y^* satisfies

$$1 + r^{be}(y^*) = \frac{1 - q(b - F)}{1 - q} \tag{11}$$

Finally, set $r^*(y) = \frac{1-q(b-F)}{1-q}$ for each $y < y^*$ and $r^*(y) = r^{be}(y)$ for each $y \geq y^*$.

Economies for which y^* is interior are shown on Figures 5(b) and 5(c).[21] For reasons that will become obvious below, we will refer to borrowers with income below y^* as belonging to the *risk-shifting region* of the income distribution.

Next, it will be useful to introduce two types of banks: banks specialized in safe lending (*safe banks* for brevity) and banks specialized in risk-shifting (*risky banks*). Safe banks have zero default probability. Risky banks default with probability q, which is the ex-ante probability of the bad aggregate state.

Proposition 2. *(Sorting of banks) The equilibrium is characterized by a complete sorting of banks into safe and risky. Each risky bank lends only to borrowers with income below y^* whereas each safe bank lends only to borrowers with income greater than or equal to y^*.*

Since all banks are ex-ante identical, each must be indifferent between specializing in risk-shifting or in safe lending. We refer to this situation as the *competition to risk shift*. Stated differently, since each bank has the (real) option to begin risk shifting, and all banks are ex-ante identical, the equilibrium value of this real option will be driven to zero.

The sorting of banks into safe and risky emerges endogenously to satisfy the demand for credit originating in the risk-shifting region of the income distribution. Specifically, the credit volume originating from the borrowers with income below y^* equals

$$\mu \equiv P_0 \int_{\underline{y}}^{y^*} \eta(y) h(y) dy. \tag{12}$$

Proposition 2 implies the equilibrium mass of risky banks is proportional to the credit volume originating from borrowers with income below y^*.

[21] One can show that $r^{be}(y)$ is strictly decreasing in y. Hence, the solution to (11), when it exist, is also unique.

Corollary 1. *The equilibrium mass of risky and safe banks equals* $P_0 \int_{\underline{y}}^{y^*} \eta(y)h(y)dy$ *and* $P_0 \int_{y^*}^{\overline{y}} \eta(y)h(y)dy$ *respectively.*

Corollary 1 implies that the proportion of safe relative to risky banks is dictated by the relative demand for credit originating among high and low income segments of the population. In other words, the distribution of income is directly related to the fraction of banks engaged in risk shifting.

To summarize: the competition to risk-shift leads to negative NPV loans to borrowers with income below an endogenous cutoff y^* (Proposition 1) and bank sorting into safe and risky (Proposition 2). This sorting, in turn, adjusts to satisfy on the demand for credit originating in different regions of the income distribution (Corollary 1).

5.3 How does income inequality shape bank risk

In this section, we analyze how varying the income distribution shapes the measure of banks specialized in risk-shifting. Since we want to focus on the effect of inequality we fix the mean income level to one, $\int_{\underline{y}}^{\overline{y}} yh(y)dy = 1$, and analyze the effect of a mean-preserving spread of the income distribution.[22]

The income distribution $\tilde{H}$ (with p.d.f. $\tilde{h}$) denotes a mean-preserving spread of the income distribution H. Let $\tilde{y}^*$ denote the cutoff corresponding to $\tilde{H}$. Then the mass of risky banks corresponding to $\tilde{H}$ is

$$\tilde{\mu} \equiv \tilde{P}_0 \int_{\underline{y}}^{\tilde{y}^*} \eta(y)\tilde{h}(y)dy.$$

where $\tilde{P}_0$ is the corresponding house price given in (8).

It will be useful to distinguish between the *direct* channel and the *indirect* channel of inequality. The direct channel operates by determining the fraction of households with income below y^*. The indirect channel operates by affecting the location of the cutoff y^*. We begin with the direct channel.

[22]Focusing on mean-preserving spreads allows us to isolate the effect of income inequality and to show that it arises separately from changes in the mean level of income. Note that the two channels outlined in this section would emerge whenever we compare one income distribution to another.

Proposition 3. *(The direct effect of inequality) Fix y^* and suppose that $\tilde{H}$ is a mean-preserving spread of H then*

(i) If $\tilde{h}(y) > h(y)$ for any $y \in [\underline{y}, y^*]$ then $\tilde{\mu}^* > \mu^*$

(ii) If $\tilde{h}(y) < h(y)$ for any $y \in [\underline{y}, y^*]$ then $\tilde{\mu}^* < \mu^*$.

Depending on the initial location of the cutoff y^*, more unequal distribution of income can either pull more households into the risk shifting region as in case *(i)* (as depicted on Figure 6(b)) or pull more households away from the risk shifting region as in case *(ii)* (as depicted on Figure 6(c)).

The indirect effect of inequality affects the location of the cutoff y^* through its effect on the house price P_0. Specifically, the cutoff is an increasing function of the house price

$$\frac{\partial y^*}{\partial P_0} > 0 \tag{13}$$

The reason is straightforward: higher house price implies that home-owners become more indebted, and therefore more likely to default in the bad state. Higher rate of default leads to a higher cutoff y^*, which pushes more borrowers into the risk shifting region of the income distribution, and therefore, more banks end up specializing in risk-shifting.

Next, the equilibrium house price is proportional to the overall housing stock.

$$P_0 = c_0 + c_1 \int_{\underline{y}}^{\overline{y}} \eta(y)h(y)dy \tag{14}$$

The house price depends on the income distribution $h(y)$, the home-ownership rates among different income groups $\eta(y)$ and the technological parameters (c_1, c_2) governing the housing construction cost.

The next result a a straightforward application of the properties of mean preserving spreads.

Proposition 4. *(The indirect effect of inequality) Suppose $\tilde{H}$ is a mean preserving spread of H then if the home-ownership rate $\eta(y)$ is a convex (concave) function of y we have $y^* < \tilde{y}^*$ $(y^* > \tilde{y}^*)$.*

Stated in terms of demand elasticity: if the income-elasticity of housing demand is greater (less) than one then holding mean income fixed, but increasing the proportion of high income households exerts an upward (downward) pressure on the house price level, which in turn, leads to higher (lower) y^*.

The interaction of the direct channel (Proposition 3) and the indirect channel (Proposition 4) in shaping bank risk is illustrated on Figures 5(b) and 5(c). Figure 5(b) depicts an economy in which higher inequality is associated to a larger fraction of risk-shifting banks. This occurs because both the direct channel $\tilde{h}(y) > h(y)$ for $y < y^*$ and the indirect channel $y^* < \tilde{y}^*$ contribute to a larger fraction of banks engaged in risk-shifting. Figure 5(c), in contrast, depicts an economy in which higher inequality is associated to a lower fraction of risk-shifting banks. That is, both the direct channel $\tilde{h}(y) < h(y)$ for $y < y^*$ and the indirect channel $y^* > \tilde{y}^*$ contribute to a fewer fraction of banks engaged in risk-shifting.

Before proceeding to the section where we connect the model to the data, there is one important question we have not addressed yet. What happens when risk shifting is not possible? We take up this issue in the next section.

5.4 Back to Keeley: what if banks cannot risk shift

We now show that neither the direct nor the indirect channel through which the distribution of income shapes bank risk will emerge unless banks can also shift risk to their creditors. In the process we tie back the discussion to the *Keeley's observation* highlighted in the Introduction, namely why do banks allow their bankruptcy risk to increase.

Proposition 5. *Suppose risk-shifting is not possible (or prohibitively costly), then bank default is not consistent with equilibrium and the distribution of income will have no effect on bank risk.*

The intuition is the following. At the margin, any bank trades-off the gain from increased risk taking with the losses from the increased risk of losing its *franchise value.* If a bank cannot shift risk to their creditors, then the only way to get compensated for the possible loss of its franchise is by charging higher lending rate than what is necessary to break-even on its loan book.

However, with free-entry into banking, this situation cannot be sustained since new bank competitors will enter, out-compete the incumbent by charging lower loan rates, and still make a positive profit. As a result, in equilibrium, banks do not take excessive risk. In other words, even though bankers are risk neutral, competition and free entry implies that they behave as though they are risk averse.

Relation to models of rational bubbles. The relation between the distribution of income and bank risk in this model is driven by the competition to risk-shift mechanism. If risk-shifting is not possible, then bank risk will not emerge in this setup. The competition to risk shift mechanism is related to Allen and Gale (2000) model of rational bubbles. In their model traders protected by limited liability bid up the price of a risky asset above its fundamental value.

In our model, the banks play the role of the traders and the rational bubble takes the form of below break-even interest rate for borrowers with income below y^*. Unlike Allen and Gale, however, the supply of the risky asset is determined in equilibrium through the location of the cutoff y^* and the fraction of borrowers who belong to the risk shifting region. That is, through the interaction of the direct and the indirect channel of inequality described in Propositions 3 and 4.

6 Using the model to understand the data

In this section we argue that the model offers a parsimonious way to understand the data patterns documented in Section 3. In order to map the model to the data, a couple remarks are in order. First, the equilibrium should be interpreted as describing the outcome in the long-run after banks have had sufficient time to adjust their portfolios and capital holdings. Second, since the data exercise in Section 3 was at a regional level, the equilibrium of the model should be thought of as describing the outcome within a given region, such as Metropolitan Statistical Area. Third, each bank is interpreted as operating within one region only.[23]

[23] The effect of national banks is described later in this section.

6.1 Inequality and the dispersion of bank risk

The model implies that the dispersion of bank risk is an equilibrium consequence of the competition to risk shift. More unequal economies will be characterized by greater sorting of banks. Figure 5(d) plots the standard deviation of bank risk as a function of the Gini coefficient for an economy with low bank franchise values (the solid line) and for an economy with high bank franchise values (the dashed line) economy. Specifically, each bank has a franchise value of 0.05 (as a ratio of its total assets) in the economy represented by the solid line and a franchise value of 0.2 (as a ratio of its total assets) in the economy represented by the dashed line.

In both economies, the probability of the bad state is 10 percent and, conditional on the bad state, the drop in the house price is assumed to be 40 percent. In addition, each bank must operate with a minimum capital ratio of 10 percent. As we vary the Gini coefficient from 0.35 to 0.55 the standard deviation of bank risk varies from about 0.02 to about 0.027 in the low franchise value economy and from about 0.018 to about 0.026 in the high franchise value economy. Moreover, for each value of the Gini coefficient, the dispersion of bank risk is greater in the low franchise value economy. For comparison, as we vary the Gini from 0.35 to 0.55 the standard deviation of the banks predicted probabilities of default varies from about 0.01 to 0.02 in the data. Finally, the effect of inequality on the dispersion of bank risk is economically significant since, as we vary the Gini coefficient, the average probability of default remains below one percent both in the model and in the data.

Thus, although highly stylized the prediction of the model closely matches with the empirical patterns on Figure 4, namely, higher income inequality is associated to grater dispersion of bank risk. In addition, observe that the low franchise value economy can be interpreted as emerging as a result of increased competition from outside financial institutions i.e. those whose business is not predominantly located in a single Metropolitan Statistical Area. We will revisit this observation in the section on the effect of competition.

6.2 Inequality and average bank risk

The equilibrium co-existence of safe and risky banks implies that rising inequality tends to increase the dispersion of bank risk, whereas at the same time, the effect on the average level of bank risk remains ambiguous. For example, both the average level and the dispersion of bank risk increases as we move from an economy with low to an economy with high inequality as shown on Figure 5(b). On the other hand, while the dispersion of bank increases, the mean level of bank risk decreases as we move from an economy with low to an economy with high inequality on Figure 5(c).

The relation between the distribution of income and the distribution of bank risk can be illustrated in a straightforward way through the following examples. Suppose the income distribution is binary: fraction of the population has high-income, the remaining fraction has low-income. We will compare to otherwise identical economies: one with perfect equality $y_L = y_H$ and the other with some inequality $\tilde{y}_L < \tilde{y}_H$. We assume that the mean level of income in both economies is the same and equal to one.

First, assume $y^* > 1$ and $\tilde{y}_L < \tilde{y}^* < \tilde{y}_H$. Proposition 2 implies that all banks in the economy with perfect equality are safe since all borrowers are outside of the risk-shifting region. Thus, both the mean level and the dispersion of bank risk in this economy equal zero. On the other hand, Proposition 2 implies that the banking sector in the more unequal economy is characterized by sorting into safe and risky banks. All risky banks specialize in credit to low-income households. All safe banks, at the same time, specialize in credit to high-income households.

Second, assume $y^* < 1$ and $\tilde{y}_L < \tilde{y}^* < \tilde{y}_H$. Proposition 2 now implies that all banks in the economy with perfect equality are risky since all borrowers belong to the risk-shifting region. On the other hand, only low-income borrowers belong to the risk shifting region in the more unequal economy. Proposition 2 then implies that fraction of the banks in the high inequality economy will be safe and specialize in lending to high-income borrowers, whereas the remaining fraction will be risky.

Summarizing: high inequality leads to higher dispersion and higher average bank risk in the first example. In the second example, in contrast, high inequality leads to higher dispersion but lower average level of bank risk. Thus, in both cases, the dispersion of bank risk is positively related to income inequality, but the effect on the mean level of bank risk is ambiguous and it depends on the starting point.

This is consistent with what we find in the data as described in Section 3. Namely, there is a clear and statistically significant positive relation between income inequality and the dispersion of bank risk. No such relation exist between income inequality and the mean level of bank risk. In other words, one must go beyond first moments in order to fully characterize the effect of inequality on bank risk.

6.3 The effect of competition

The model focuses on regional banking markets. At the same time, greater competition from outside financial institutions can amplify the sorting of the regional banking markets though at least three channels. In this section, we show how the effect of outsider banks can be captured in a reduced form by examining comparative statics.

First, suppose we model increased competition from outsider banks as regional banks having to pay higher deposit rate r_d. The effect of higher deposit rate is to increase the cutoff y^* and expand the risk-shifting region as a result.

Second, more competition from outsider banks could undermine the local banks' traditional business model. We can capture this situation through lower return on storage (if storage is interpreted as capturing local banks' core business activities). Lower return on storage leads to higher cutoff y^*, and therefore, to a greater proportion of regional banks specialized in risk shifting.

Third, a reduction in entry barriers to a regional market would attract outside competitors and undermine local banks' franchise values. Lower franchise values are associated to larger cutoff y^*, and therefore, a grater fraction of potential borrowers belonging to the risk-shifting region. This is illustrated on Figure 5(d) where the sorting of banks into safe and risky, and the associated dispersion of bank risk, is greater in the high-competition, low-franchise value economy (represented by the the solid line).

These three channels are not mutually exclusive and can operate simultaneously leading to greater dispersion of bank risk in more competitive banking regions while, just as before, the effect on the mean level of bank risk remains ambiguous. The second implication, namely the ambiguous effect on mean bank risk, is consistent with the mixed empirical evidence presented in the broader empirical literature on the effect of greater banking competition on the overall level of financial stability with one set of studies finding a positive relation (see, e.g., Jiménez et al., 2013; Braggion et al., 2017; Gissler et al., 2019), whereas another finding a negative relation (see, e.g., Jayaratne and Strahan, 1998; Schaeck et al., 2009; Carlson and Mitchener, 2009). One way to account for the mixed empirical findings is through the competition to risk shift and the associated equilibrium sorting of banks into different risk categories.[24]

7 Conclusion

We presented empirical patters linking income inequality to bank risk. Regions in the U.S. with more unequal income distribution tend to have greater dispersion of bank risk. We proposed a model to explain the empirical patterns as an equilibrium outcome. The model assumes deposit insurance and relies on one friction only: risk shifting. The core mechanism is based on Allen and Gale (2000) model of rational bubble generated by limited liability applied to the banking context.

Banks issue insured deposits and compete to attract risky borrowers by cutting lending rates until loans to high-risk low-income borrowers are issued at below break-even interest rates. We referred to this situation as the competition to risk-shift. In equilibrium, each bank is indifferent between specializing in risk shifting and remaining safe. However, risky and safe banks follow different lending strategies and their clientele do not overlap. This sorting of banks, implies that the proportion of high and low risk banks is driven by the distribution of income.

Specifically, the equilibrium fraction of banks specializing in risk shifting adjusts to satisfy the demand for credit originating in the high-risk credit segments. Simultaneously, the equilibrium fraction of safe banks adjusts to satisfy the demand for credit originating in the low-risk credit segments. In other words, safe and risky banks co-exist and focus

[24]It is also theoretically possible that outsider banks have a comparative advantage in lending to high-risk borrowers due to improved screening and monitoring, diversification effects or implicit government guarantees. If this is the case, then lower entry barriers would increase the stability of the regional banking market since the new entrants end up absorbing the risky borrowers even if the franchise values of the local banks decreases as they face increased competition.

on different types of borrowers.

The model provides a simple way to account for patterns we find in the data. The equilibrium sorting of banks into safe and risky will be more pronounced - and the dispersion of bank risk greater – in regions with more unequal distribution of income. At the same time, the effect on the overall level of bank risk is not so clear cut and it depends on the relative size of the high-risk and the low-risk credit segments.

The competition to risk shift also implies that low-income households obtain credit at below break-even interest rates from banks engaged in risk shifting. This situation can be interpreted as lax lending standards leading to subprime lending boom. One implication of the model is that regions with greater housing boom and subsequent bust are also characterized by more pronounced sorting of banks into safe and risky, and therefore, greater dispersion of bank risk.

The model assumed that the risk of default is monotonically decreasing in income. This assumption understates the effect of income inequality on bank risk since it ignores speculative mortgage demand (i.e. second-home mortgages) which is more prevalent among high-income households. This feature can be incorporated into the model in straightforward way and it would imply that risky shifting banks specialize in loans to low-income households and in speculative mortgage credit.

Finally, the distribution of income will not shape bank risk in this environment unless banks can engage in risk shifting behavior. If risk shifting can be made prohibitively costly through regulation (such as risk-weighted capital ratios) the distribution of income and the distribution of bank risk will not be related. In this sense, the model is based on implicit regulatory forbearance which makes risk-shifting a viable opportunity for depository institutions. This form of regulatory forbearance can be especially costly in regions with rising income inequality.

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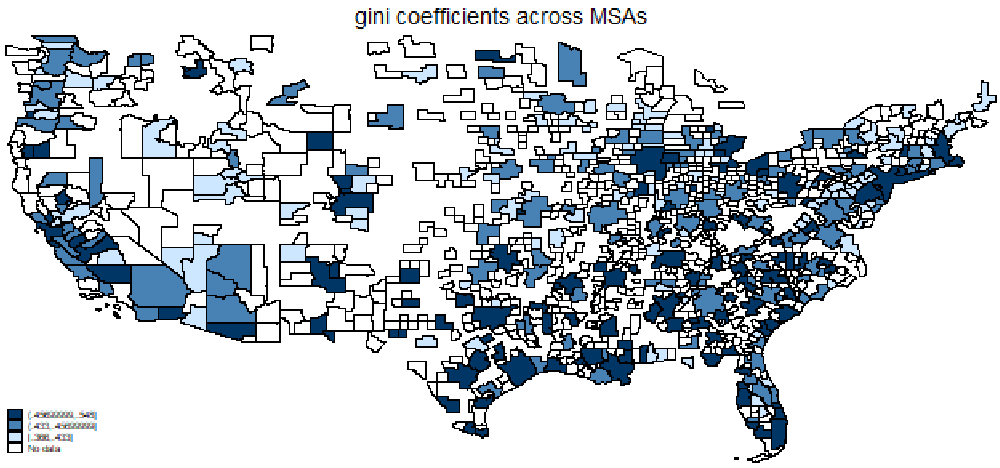
Appendix

A. Figures

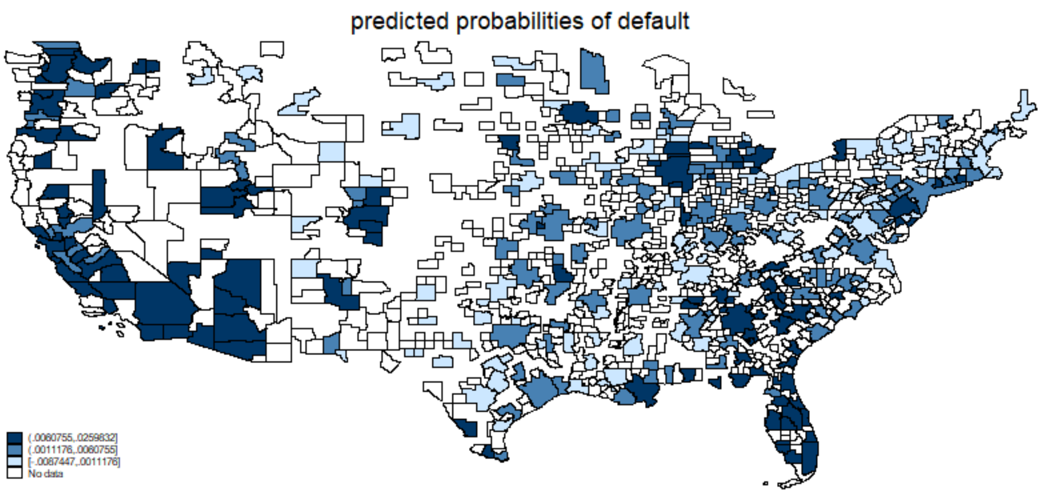
Figure 1: Income inequality across Metropolitan Statistical Areas



(a) Gini coefficients

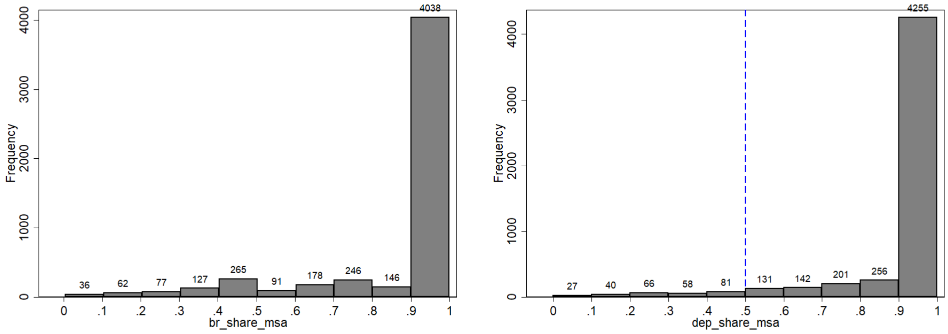
This figure shows the Gini coefficients per MSA for the year 2008 (source: U.S. Census Bureau/ American Community Survey). Darker colors represent higher coefficients, i.e., higher inequality. Regions that are shown in white colors are not part of an MSA, or the data on the Gini coefficient is not available.

Figure 2: Bank risk across Metropolitan Statistical Areas



This figure shows banks' average predicted probabilities of default per MSA. Darker colors represent higher predicted probabilities of default, i.e., higher bank risk. Regions that are shown in white colors are not part of an MSA, or the data on bank risk is not available.

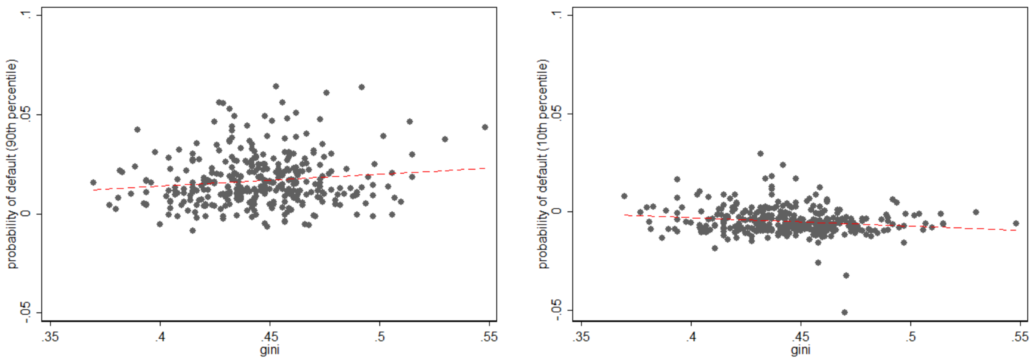
Figure 3: Regional concentration of banks



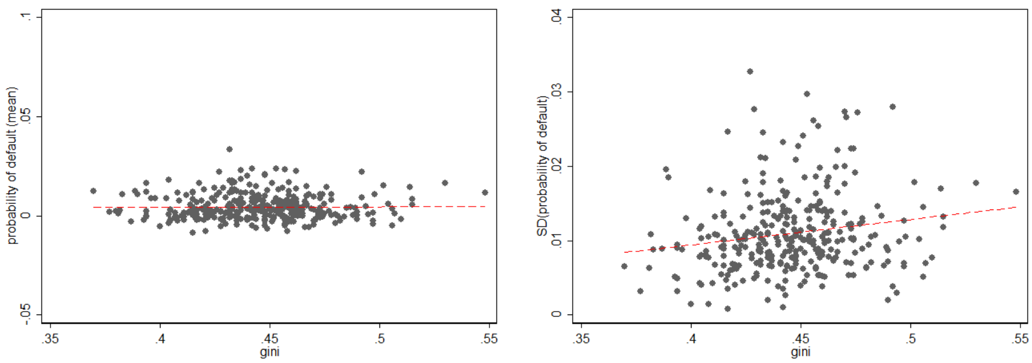
(a) branches (b) deposits

This figure shows the share of branches (left graph) and deposits (right graph) of each bank that are located in the same MSA as the bank's headquarters in the year 2000. For our main sample, we focus on banks with a respective share of deposits of 50% or more, which includes all banks to the right of the horizontal line in the right graph. The idea of this requirement is to exclude large national banks, such as Bank of America, from the sample.

Figure 4: First graphical evidence



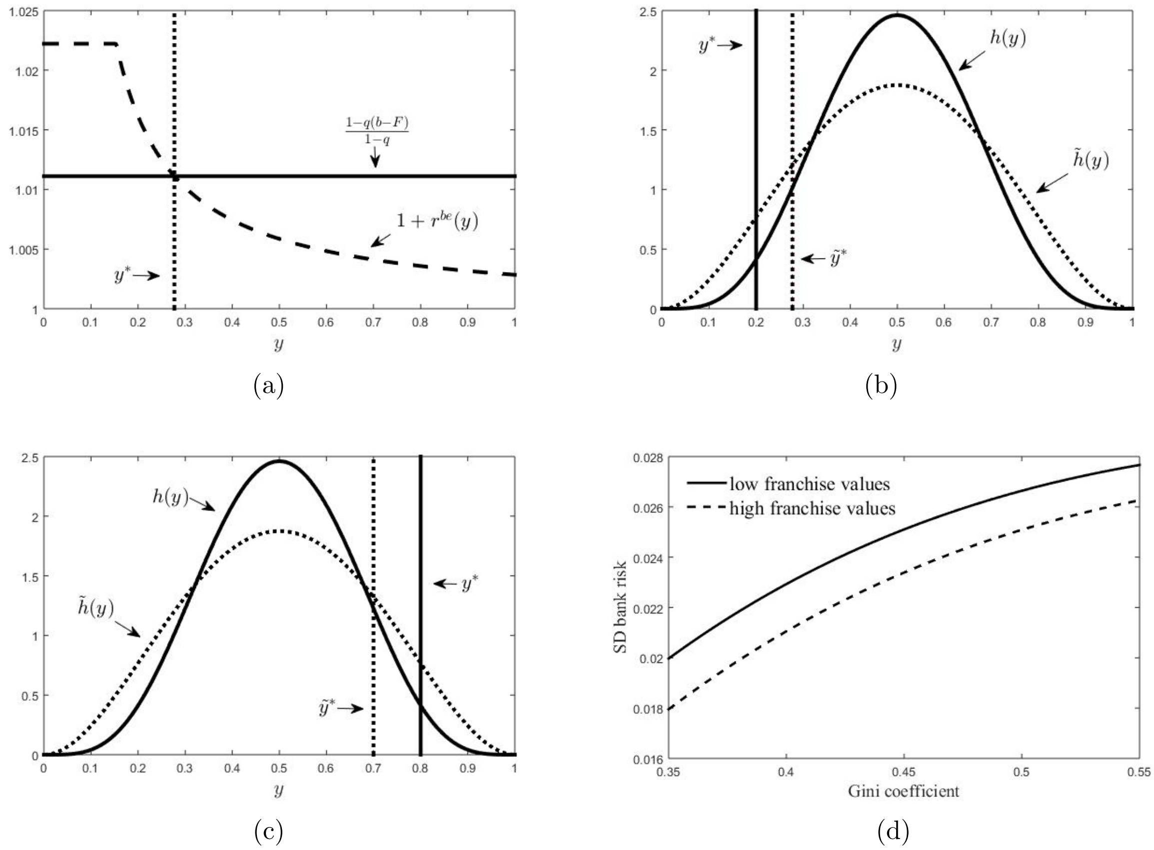
(a) most risky banks per MSA (P90_PD) (b) least risky banks per MSA (P10_PD)



(c) average bank risk per MSA (mean PD) (d) dispersion of bank risk per MSA (SD_PD)

This figure shows the relation between income inequality (Gini coefficient) and different measures of bank risk per MSA.

Figure 5: Determination of equilibrium and comparative statics.



This figure shows the determination of equilibrium and comparative statics as discussed in Section 5.

B. Main Tables

B.1 Descriptives

Table 1: Variable description

Variable name	Description
Panel A: Bank characteristics	
PD	Banks' predicted probabilities of default, as described in detail the Online Appendix.
Failed	A dummy variable with a value of 1 if the bank failed, and 0 otherwise. Source: FDIC failed bank list.
Panel B: Banking market characteristics on the MSA level	
P10_PD	**Bank risk of least risky banks:** The long-term average per MSA of the 10th-percentile of banks' predicted probabilities of default for each MSA and year. Source: Own calculations based on FDIC data.
P90_PD	**Bank risk of most risky banks:** The long-term average per MSA of the 90th-percentile of banks' predicted probabilities of default for each MSA and year. Source: Own calculations based on FDIC data.
Mean_PD	**Average bank risk:** The long-term average per MSA of the mean of banks' predicted probabilities of default for each MSA and year. Source: Own calculations based on FDIC data.
SD(PD)	**Dispersion of bank risk:** The long-term average per MSA of the standard deviation of banks' predicted probabilities of default for each MSA and year. Source: Own calculations based on FDIC data.
Panel C: Regional characteristics on the MSA level	
Gini	The Gini-coefficient is defined as "the difference between the Lorenz curve (the observed cumulative income distribution) and the notion of a perfectly equal income distribution." A measure of 1 indicates perfect inequality, i.e., one household having all the income and rest having none. A gini measure of 0 indicates perfect equality, i.e., all households having an equal share of income. Source: U.S. Census Bureau, 2008 American Community Survey (Table B19083). Note: 2008 is the first year when this data is available on the MSA level.
Mean household income	Stated in USD 000. Source: U.S. Census Bureau, 2005 American Community Survey (Table DP03). Note: 2005 is the first year when this data is available on the MSA level.
Median household income	Stated in USD 000. Source: U.S. Census Bureau, 2005 American Community Survey (Table DP03). Note: 2005 is the first year when this data is available on the MSA level.

Table 2: Descriptive statistics

	Obs.	**Mean**	**SD**	**Min**	**P10**	**P50**	**P90**	**Max**
Panel A: Variables on the bank level								
PD	68,129	0.0046	0.0205	-0.18	-0.01	-0.00	0.02	0.35
Failed	68,129	0.0479	0.2136	0.00	0.00	0.00	0.00	1.00
Panel B: Variables on the MSA level								
P10_PD	325	-0.0047	0.0076	-0.05	-0.01	-0.01	0.00	0.04
Mean_PD	325	0.0046	0.0069	-0.01	-0.00	0.00	0.01	0.04
P90_PD	325	0.0169	0.0126	-0.01	0.00	0.01	0.03	0.07
Gini	325	0.4454	0.0278	0.37	0.41	0.45	0.48	0.55
Mean household income (000)	325	56.8740	9.9009	36.55	46.21	55.50	68.67	116.05
Median household income (000)	325	43.7629	7.7890	24.50	34.84	42.79	53.49	76.48

This table shows descriptive statistics. Variables on the bank level are not included directly in the regression analysis, but used to calculate the respective variables on the MSA level.

Table 3: Cross-correlation table

to be completed

B.2 Main regression results: MSA data in the cross section

Table 4: Bank risk of most risky banks per MSA

This table shows regression results for the empirical model presented in Section 3. See Table 1 for a detailed explanation of every variable, and Table 2 for descriptive statistics. P-values are reported in parentheses. The ***, ** and * indicate significant coefficients at the 1%, 5%, and 10% levels, respectively.

	(1) P90_PD	(2) P90_PD	(3) P90_PD	(4) P90_PD	(5) P90_PD	(6) P90_PD
gini	0.0667***	0.0646**	0.0676***	0.0500*	0.0827***	0.0683***
	(0.0084)	(0.0168)	(0.0061)	(0.0649)	(0.0009)	(0.0093)
mean_household_income			0.0002***	0.0003***		
			(0.0012)	(0.0026)		
median_household_income					0.0003***	0.0004***
					(0.0025)	(0.0027)
State FE	No	Yes	No	Yes	No	Yes
No. of MSAs	325	325	325	325	325	325
No. of banks	5453	5453	5453	5453	5453	5453
Obs. (total)	325	325	325	325	325	325
Obs. (w/o singletons)	325	322	325	322	325	322
Adj. R2	0.0185	0.2208	0.0465	0.2511	0.0419	0.2514
Within R2	0.0215	0.0199	0.0524	0.0614	0.0478	0.0618

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Table 5: Bank risk of least risky banks per MSA

This table shows regression results for the empirical model presented in Section 3. See Table 1 for a detailed explanation of every variable, and Table 2 for descriptive statistics. P-values are reported in parentheses. The ***, ** and * indicate significant coefficients at the 1%, 5%, and 10% levels, respectively.

	(1) P10_PD	(2) P10_PD	(3) P10_PD	(4) P10_PD	(5) P10_PD	(6) P10_PD
gini	-0.0212	-0.0298*	-0.0212	-0.0224	-0.0204	-0.0316*
	(0.1721)	(0.0860)	(0.1726)	(0.1833)	(0.2164)	(0.0831)
mean_household_income			0.0000	-0.0001**		
			(0.7377)	(0.0198)		
median_household_income					0.0000	-0.0002**
					(0.8227)	(0.0270)
State FE	No	Yes	No	Yes	No	Yes
No. of MSAs	325	325	325	325	325	325
No. of banks	5453	5453	5453	5453	5453	5453
Obs. (total)	325	325	325	325	325	325
Obs. (w/o singletons)	325	322	325	322	325	322
Adj. R2	0.0030	0.2504	0.0003	0.2715	0.0001	0.2691
Within R2	0.0060	0.0124	0.0064	0.0438	0.0062	0.0406

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Table 6: Average bank risk per MSA

This table shows regression results for the empirical model presented in Section 3. See Table 1 for a detailed explanation of every variable, and Table 2 for descriptive statistics. P-values are reported in parentheses. The ***, ** and * indicate significant coefficients at the 1%, 5%, and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
	Mean_PD	Mean_PD	Mean_PD	Mean_PD	Mean_PD	Mean_PD
gini	0.0121	0.0076	0.0125	0.0051	0.0195	0.0083
	(0.3982)	(0.5663)	(0.3720)	(0.6919)	(0.1794)	(0.5279)
mean_household_income			0.0001***	0.0000		
			(0.0075)	(0.3080)		
median_household_income					0.0001**	0.0001
					(0.0160)	(0.2747)
State FE	No	Yes	No	Yes	No	Yes
No. of MSAs	325	325	325	325	325	325
No. of banks	5453	5453	5453	5453	5453	5453
Obs. (total)	325	325	325	325	325	325
Obs. (w/o singletons)	325	322	325	322	325	322
Adj. R2	-0.0008	0.4226	0.0187	0.4236	0.0150	0.4244
Within R2	0.0023	0.0013	0.0248	0.0067	0.0211	0.0080

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Table 7: Dispersion of bank risk

This table shows regression results for the empirical model presented in Section 3. See Table 1 for a detailed explanation of every variable, and Table 2 for descriptive statistics. P-values are reported in parentheses. The ***, ** and * indicate significant coefficients at the 1%, 5%, and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
	SD_PD	SD_PD	SD_PD	SD_PD	SD_PD	SD_PD
gini	0.0358***	0.0391***	0.0360***	0.0298**	0.0420***	0.0412***
	(0.0004)	(0.0013)	(0.0004)	(0.0136)	(0.0001)	(0.0006)
mean_household_income			0.0001**	0.0002***		
			(0.0111)	(0.0001)		
median_household_income					0.0001**	0.0002***
					(0.0167)	(0.0001)
State FE	No	Yes	No	Yes	No	Yes
No. of MSAs	325	325	325	325	325	325
No. of banks	5453	5453	5453	5453	5453	5453
Obs. (total)	324	324	324	324	324	324
Obs. (w/o singletons)	324	321	324	321	324	321
Adj. R2	0.0268	0.1517	0.0477	0.2108	0.0436	0.2102
Within R2	0.0298	0.0320	0.0536	0.1027	0.0495	0.1021

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C. Predictions of banks' probabilities of default[25]

Data The data sources that we use for the prediction of the banks' default probabilities are the *Federal Deposit Insurance Corporation* (FDIC) for all bank financial data and information about bank failures.[26] The sample includes yearly data on 11,484 U.S. banks from 2000 to 2018, which results in a total of 150,856 observations. We require that a bank has its headquarters anywhere in the contiguous United States and has non-missing information for all variables we use in the analysis. See Table 8 for a description of all variables.

The number of bank failures for this sample is 571. It includes final bank failures (e.g., Washington Mutual Bank) as well as assistance transactions (e.g., in the case of Bank of America and Citigroup), as provided by the FDIC's *Bank Failures and Assistance Data* list.

Model We predict banks' probabilities of default (PD) using the following linear probability model:[27]

$$\begin{aligned} Fail_{i,t} = {} & \tau_t \times \gamma_s + \beta_1 AGE_{i,t-1} + \beta_2 CIR_{i,t-1} + \beta_3 COI_{i,t-1} + \beta_4 EQ_{i,t-1} \\ & + \beta_5 FOR_{i,t-1} + \beta_6 IENC_{i,t-1} + \beta_7 LIQ_{i,t-1} + \beta_8 LOA_{i,t-1} + \beta_9 NPA_{i,t-1} \\ & + \beta_{10} RE_{i,t-1} + \beta_{11} ROA_{i,t-1} + \beta_{12} SIZE_{i,t-1} + \epsilon_{i,t}. \end{aligned}$$

The dependent variable $Fail_{i,t}$ is a binary variable with a value of one if bank i fails in year t, and zero otherwise. The variables $\tau_t \times \gamma_s$ cover year-state fixed effects to capture developments over time in the different U.S. states. In line with the literature, we choose the first lag of all right-hand-side variables.

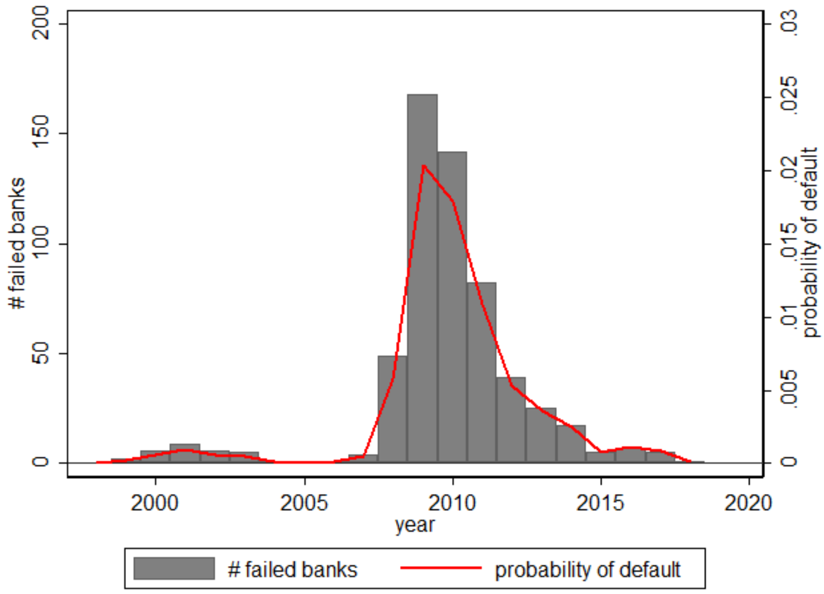
[25] This section is similar to a section in the Online Appendix of the paper *Natural Disasters and Bank Stability* by Noth and Schüwer (SAFE Working Paper, 2018).

[26] See the webpage *FDIC Bank Data & Statistics* (https://www.fdic.gov/bank/statistical/) and the webpage *Failed Banks* (https://www.fdic.gov/bank/individual/failed/).

[27] A linear probability model allows us to include year-state fixed effects. With a nonlinear probability model, the introduction of many fixed effects leads to i) practical problems because the presence of many variables makes the estimation much more difficult, and ii) the incidental parameters problem (Greene et al., 2002; Fernandez-Val, 2009).

Results Regression results of the probability model are shown in Table 9. Figure 6 illustrates the banks' average predicted probabilities of default per year, which increase significantly during the 2008 financial crisis. Predicted probabilities of default (PD) are then used as a measure of bank risk for the regressions in the main part of this paper.

Figure 6: Failed banks and probabilities of default



This figure shows the number of failed banks (source: *Bank Failures and Assistance Data* from the FDIC webpage) as ell as the banks' average predicted probabilities of default (own predictions, as described in this section).

Table 8: Predictions of default probabilities/ variable description

Variable name	Description
AGE	**Age:** Banks' age as the natural logarithm of the quarterly distance to each bank's date of establishment. Source: FDIC ($\ln(qtr - birthqtr)$).
CIR	**Cost-to-income ratio:** The ratio of banks' total cost to income. Source: FDIC ($nonix/(nim + nonii)$).
COI	**Commercial and industrial loan ratio:** The ratio of banks' commercial and industrial loans to total assets. Source: FDIC ($lnci/asset$).
EQ	**Equity ratio:** The ratio of total equity to total assets. Source: FDIC ($eqv/100$).
FAIL	**Bank failure:** Bank failures come from the FDIC's *failed bank list* (transaction types PA, PI, PO, PI). Source: FDIC (https://www.fdic.gov/bank/individual/failed/). To account for public bailouts, we include "technical" bank failures if a bank's sum of equity and reserves is lower than half of its non-performing assets (see, Cole and White, 2010).
FOR	**Foreclosure ratio:** The ratio of a bank's other real estate owned, which is not directly related to its business and consists largely of foreclosed property, to total assets. Source: FDIC ($ore/asset$).
IENC	**Income earned, not collected on loans:** The ratio of banks' income not collected on loans to total assets. Source: FDIC ($oaienc/asset$).
LIQ	**Liquidity:** The ratio of difference between federal funds purchased and sold to total assets. Source: FDIC ($(frepp - frepo)/asset$).
LOA	**Gross loan ratio:** The ratio of banks' gross loans to total assets. Source: FDIC ($lnlsgr/asset$).
NPA	**Non-performing assets ratio:** The sum of loans past due 30-90+ days but still accruing interest and nonaccrual loans, scaled by total assets. Source: FDIC ($(p9asset + p3asset + naasset)/asset$).
RE	**Real estate loan ratio:** The ratio of banks' real estate loans to total assets. Source: FDIC ($lnre/asset$).
ROA	**Return on assets:** Net income as a percentage of average total assets. Source: FDIC ($roa/100$).
SIZE	**Bank size:** The natural logarithm of banks' total assets. Source: FDIC ($\ln(asset)$).

Table 9: Predictions of default probabilities

Notes: The column shows results of the linear probability model. See Table 8 for a detailed description of all variables. Standard errors are clustered at the bank level. ***, ** and * indicate significant coefficients at the 1%, 5%, and 10% levels, respectively.

	(1)
	failed_yr
L.age	-0.0022***
	(0.0000)
L.cir	0.0001
	(0.1647)
L.ci	0.0000
	(0.9995)
L.eq100	-0.0383***
	(0.0000)
L.for	0.6078***
	(0.0000)
L.ienc	-0.0611
	(0.5957)
L.liq	-0.0016
	(0.4734)
L.loa	-0.0210***
	(0.0000)
L.npa	0.6603***
	(0.0000)
L.re	0.0027
	(0.2674)
L.roa100	-0.0618**
	(0.0262)
L.size	0.0012***
	(0.0000)
Constant	0.0104***
	(0.0007)
Year × State FE	Yes
Clustering	Bank level
Unique banks	11484
Failed banks	571
Obs.	150,856
No. of cluster	11,484
Adj. R2	0.0944

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D. Proof of selected propositions

The following properties will be useful for the derivations to follow. First, \$1 invested in mortgage loans to borrowers with income y yields $1 + r(y)$ in the good aggregate state and

$$\psi\left(r(y)\,|\,y,\ P_0\right) \equiv (1+r) - G\left[P_0(1+r(y)-l)\,\middle|\,y\right](1+r(y)-l)$$

in the bad aggregate state. The break-even interest rate $r^{be}(y)$ for a borrower with income y satisfies

$$(1-q)\left(1+r^{be}(y)\right) + q\psi\left(r^{be}(y)\,|\,y,\ P_0\right) = 1 \tag{15}$$

That is, \$1 invested in mortgage loans to income y borrowers yields \$1 in expectation, which is equal to the return on storage. If $y_1 < y_2$ then $G(.|y_2)$ dominates $G(.|y_1)$ in the first-order stochastic sense. Hence

$$\frac{\partial r^{be}(y)}{\partial y} < 0 \qquad \text{and} \qquad \frac{\partial \psi^{be}(y)}{\partial y} > 0 \tag{16}$$

Next, competition and free entry implies that, in equilibrium, the net expected payoff to any bank active in any mortgage market must equal the net expected payoff from holding storage. If a bank invest $b + k$ in storage, its payoff will be

$$1 - b - k$$

This observation has two implications. First, for any $r(y) < r^{nb}(y)$ we have

$$(1-q)(1+r(y)) + q\psi(r(y)\,|\,y,\ P_0) < (1-q)(1+r^{be}(y)) + q\psi(r^{be}(y)\,|\,y,\ P_0) = 1 \tag{17}$$

Otherwise, by setting an interest slightly below $r^{be}(y)$, the net expected payoff for the bank would exceed the net expected payoff from holding storage. Second, equilibrium interest rate for income-y borrowers $r^*(y)$ must satisfy

$$r^*(y) \leq r^{nb}(y) \qquad \text{for } y \in \left[\underline{y},\, \overline{y}\right]$$

That is, the equilibrium interest rate for any borrower type cannot exceed the break-even interest rate.

Proposition 1.

Proof. As described in the text, the cutoff point y^* is obtained as the solution to

$$1 + r^{be}\left(y^*\right) = \frac{1 - q(b - F)}{1 - q} \tag{18}$$

where $r^{be}(y)$ is the break-even interest rate defined in (9). From (16) we know that $r^{be}\left(y\right)$ is strictly decreasing in y. Hence, the solution to (18), when it exist, is unique. Also, the cutoff y^* is interior $\underline{y} < y^* < \overline{y}$ if and only if the following holds

$$1 + r^{be}(\underline{y}) > \frac{1 - q(1 - k - F)}{1 - q} > 1 + r^{be}(\overline{y})$$

Next, (18) and (16) imply

$$\psi\left(r^{be}\left(y^*\right) \mid y^*,\, P_0\right) = 1 - k - F \leq 1 - k$$

with strict inequality if $F > 0$. Thus, any bank specialized in credit to borrowers with income y^* becomes insolvent in the bad state. Combining these two conditions, the expected return for such a bank is

$$(1 - q)\left(1 + r^{be}\left(y^*\right) - b\right) - k - qF = 1 - b - k$$

Hence, this bank is indifferent between the safe strategy (storage) and fully specializing in risky loans to borrowers with income y^*. Next, consider any $y < y^*$. Applying (16) yields

$$\psi\left(r^{be}\left(y\right) \mid y,\, P_0\right) < \psi\left(r^{be}\left(y^*\right) \mid y,\, P_0\right) < b - F$$

That is, a bank specialized in income $y < y^*$ borrowers also becomes insolvent in the bad aggregate state. Applying (16) again, we have $r^{be}(y) > r^{be}(y^*)$, which yields

$$(1-q)\left(1+r^{be}(y)-b\right)-k-qF > 1-b-k$$

That is, a bank specialized in credit to income $y < y^*$ borrowers earns expected profits greater than storage. However, this violates the free-entry condition since the equilibrium expected profit to any to any bank cannot exceed the expected profit from holding storage. Hence, the following must hold for any $y < y^*$

$$r^*(y) = r^{be}(y^*) < r^{be}(y)$$

But since $r^*(y) < r^{be}(y)$ loans to borrowers with income below y^* carry negative net present value in equilibrium. Hence, each bank which remains solvent in the bad aggregate state strictly prefers to hold storage rather than lend to borrowers with income below y^*. □

Proposition 2.

Proof. Suppose a fraction $\alpha \in (0,1)$ of a given bank's portfolio is invested in loans to borrowers with income $y < y^*$ and the remaining fraction $1-\alpha$ is invested in loans to borrowers with income $y > y^*$. We will show that this portfolio is not consistent with equilibrium. First, since $y_1 < y^* < y_2$ the following holds

$$\psi_1 < b - F < b < \psi_2 \tag{19}$$

$$(1-q)(1+r_1-b)-k-qF = 1-b-k \tag{20}$$

$$(1-q)(1+r_2)+q\psi_2 = 1 \tag{21}$$

where, for brevity, we have defined $r_1 = r^*(y_1)$, $r_2 = r^*(y_2)$, $\psi_1 = \psi\left(r^*(y_1)\,|\,y_1,\ P_0\right)$ and $\psi_2 = \psi\left(r^*(y_2)\,|\,y_2,\ P_0\right)$. Next, define $\hat{\alpha}$ as:

$$\hat{\alpha} \equiv \frac{\psi_2 - b}{\psi_2 - \psi_1}$$

The bank remains solvent in the bad state if and only if $\alpha \leq \hat{\alpha}$. That is

$$\alpha\psi_1 + (1-\alpha)\psi_2 \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} b \qquad \text{as} \qquad \alpha \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \hat{\alpha}$$

Case 1: $\alpha \leq \hat{\alpha}$. That is, the bank is solvent in the bad state. Applying (19) and (20) yields

$$\pi_1 \equiv (1-q)(1+r_1-b) + q(\psi_1 - b) < 1-b-k$$

Whereas (21) yields

$$\pi_2 \equiv (1-q)(1+r_2-b) + q(\psi_2 - b) - k = 1-b-k$$

Hence, $\pi_1 < \pi_2$ and the bank's expected payoff is

$$\pi_{\text{mixed}} \;=\; \alpha\pi_1 + (1-\alpha)\pi_2 < 1-b-k$$

which is not consistent with equilibrium because the bank can earn greater payoff by holding storage (or by lending only to type y_2 borrowers).

Case 2: $\alpha > \hat{\alpha}$. That is, the bank is insolvent in the bad state. From (20) we have

$$\pi_1 \equiv (1-q)(1+r_1-b) - k - qF = 1-b-k$$

Whereas (19) and (21) imply

$$\pi_2 \equiv (1-q)(1+r_2-b) - k - qF < (1-q)(1+r_2-b) + q(\psi_2 - b) - k$$

Hence, $\pi_1 > \pi_2$ and the bank's expected payoff in this case is

$$\pi_{\text{mixed}} \;=\; \alpha\pi_1 + (1-\alpha)\pi_2 < 1-b-k+F$$

which again is not consistent with equilibrium because the bank can earn greater payoff by holding storage (or by lending only to type y_1 borrowers).

Hence, our assumption that a portfolio such that $y_1 < y^* < y_2$ can be part of the equilibrium outcome cannot be true since any bank following such a strategy is better off deviating to storage. As a result, each bank either lends only to borrowers with income below y^* or lends to borrowers with income greater than or equal to y^*. $\square$

Proposition 5.

Proof. Suppose there is a bank which specializes in income-y loans and defaults in the bad aggregate state. That is,

$$\psi(r(y) \,|\, y,\ P_0) < b(1 + r_d)$$

where r_d is the interest rate this bank offers its depositors. Hence, this bank is insolvent with probability q (equal to the probability of the bad aggregate state) and its expected payoff is

$$(1 - q)\,(1 + r(y) - b(1 + r_D)) - k - qF$$

where $F \geq 0$ is the cost of default uncured by the equity holders in the bank. Note that this bank loses its franchise with probability q. We show that if risk shifting is not possible, then the bank strictly prefers to hold storage. Indeed, f the bank cannot risk shift, then the return to deposits must ensure that they break-even in expectation. That is,

$$(1 - q)(1 + r_D)b + q\psi(r_y \,|\, y,\ P_0) = b$$

which implies that the interest rate on deposits r_D adjusts to satisfy

$$r_D = \frac{b - q\psi(r_y \,|\, y,\ P_0)}{1 - q} - 1$$

Plugging the above expression for r_D into the bank's expected payoff yields

$$
\begin{aligned}
(1-q)(1+r(y)-b(1+r_D))-k-qF &= (1-q)(1+r(y))+q\psi(r(y)\,|\,y,\ P_0)-(b+k+qF) \\
&\leq (1-q)(1+r^{be}(y))+q\psi(r^{be}(y)\,|\,y,\ P_0)-b-k-qF \\
&\leq 1-b-k
\end{aligned}
$$

The weak inequality on the second line follows from (17). The weak inequality on the third line follows from (15) and $F \geq 0$. Finally, we have

$$
(1-q)(1+r_y-b(1+r_D))-k-qF < 1-b-k
$$

whenever $F > 0$. Hence, this bank strictly prefers to hold storage (rather than gambling its franchise), a situation which is not consistent with equilibrium. Hence, when risk-shifting is not possible, equilibrium default does not occur among banks with strictly positive franchise values. □